Multiple Trait Selection

GENE422/522 Topic 14

Issues with MT selection

- We have to spread our selection efforts over several traits,
- Not all traits are equally important economically
- Not all traits are equally heritable
- There are correlations between traits
    - Selection for one trait gives also a correlated response for other traits

– How to weight optimally the different traits

Multiple Trait Selection

- Defining MT Selection Weights
- Prediction MT Selection Response
- Manipulating MT Selection Response

Selection Index Concept

Selection Criteria		Breeding Objective
Own performance X_1		
		Breeding Value (s)
Performance on relatives X_2		
Correlated Traits X_n		

Selection Index (multiple regression)

$$EBV = Index = b_1X_1 + b_2X_2 + b_3X_3 + \ldots\ldots + b_nX_n$$

Selecting for multiple traits in an index

Using phenotypic measurements on traits

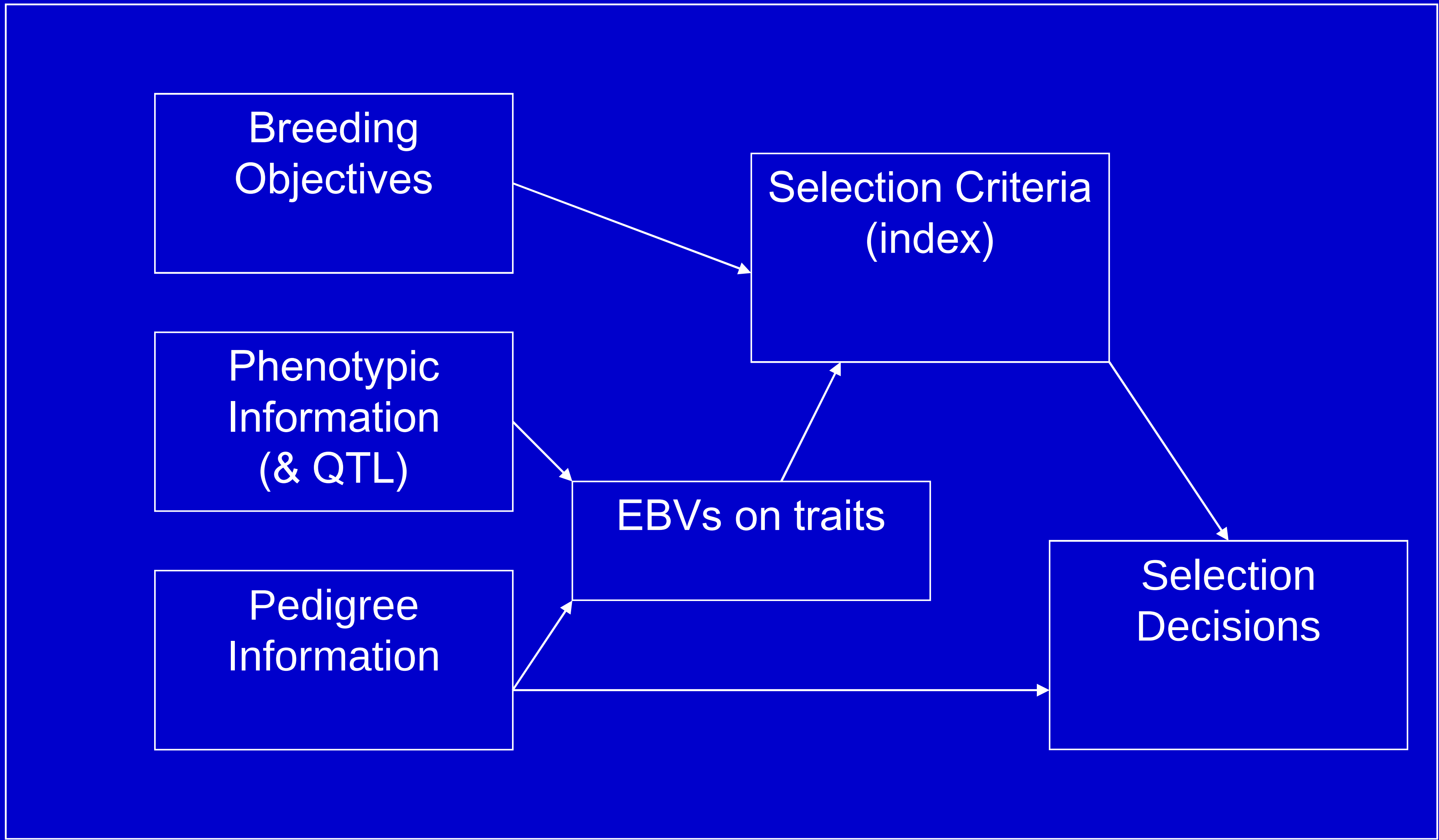
$$\text{Index} = b_1P_1 + b_2P_2 + \ldots + b_nP_n$$

Using EBVs

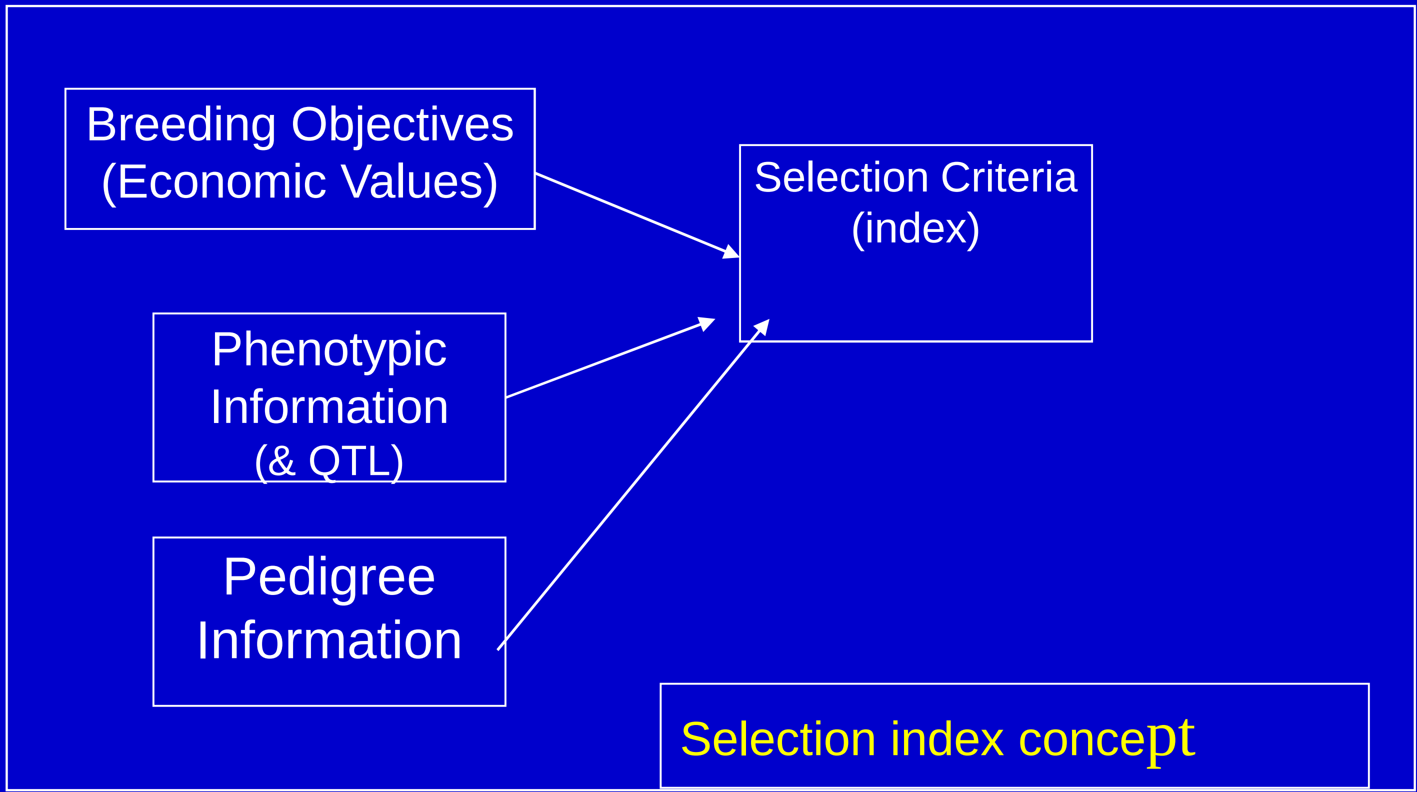
$$\text{Index} = v_1EBV_1 + v_2EBV_2 + \ldots + v_nEBV_n$$

weights (v_i) are equal to economic values!

Relationship between information, pedigree, index and multiple trait selection

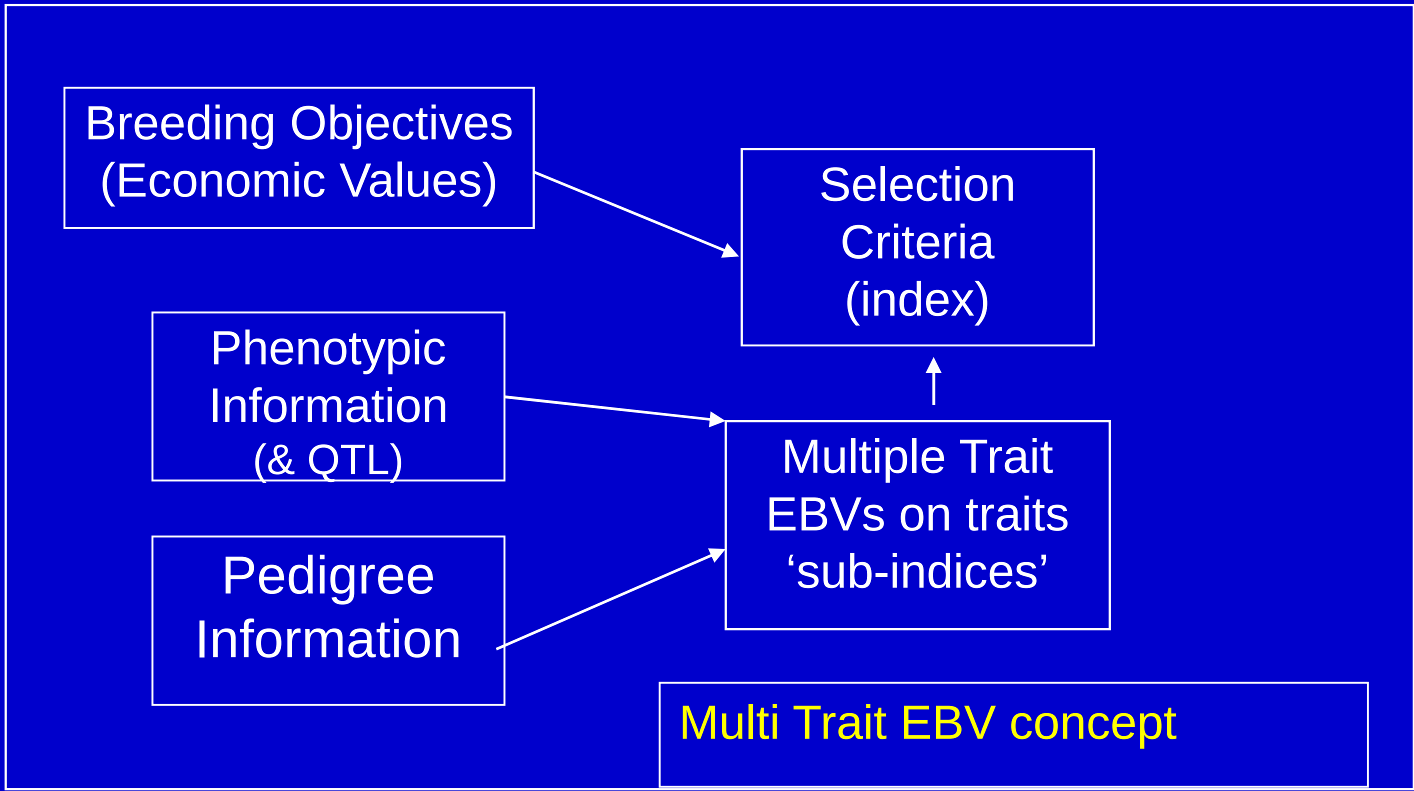


Relationship between information, pedigree, index and multiple trait selection



Index = $0.195.W_{own} + 0.056.W_{sire} + 0.164.W_{HS} - 0.916.FI_{sire}$

Relationship between information, pedigree, index and multiple trait selection



Index = $1.W_{EBV} - 10.FI_{EBV}$

Multi Trait Selection Index

- Need to combine
  - the relative economic weights
  - genetic parameters (heritabilities, correlations)

to determine the weights we put on the observed phenotypes

$$\text{Index} = b_1P_1 + b_2P_2 + \dots + b_nP_n$$

Selection index with more information sources (multiple regression)

p = vector with phenotypes (criteria)

g = breeding objective (single trait BV here)

var(p) = P = matrix = $\begin{bmatrix} \mathrm{var}(p_1) & \mathrm{cov}(p_1, p_2) \\ \mathrm{cov}(p_2, p_1) & \mathrm{var}(p_2) \end{bmatrix}$

cov(p,g) = G = vector = $\begin{bmatrix} \mathrm{cov}(p_1, g) \\ \mathrm{cov}(p_2, g) \end{bmatrix}$

weights: $b = P^{-1}G$

Selection index with more information sources and with more objective traits (multiple regression)

p = vector with phenotypes (criteria)

H = breeding objective (multiple traits here)

$= v_1 g_1 + v_2 g_2$

$$\mathrm{var}(p) = P = \text{matrix} = \begin{bmatrix} \mathrm{var}(p_1) & \mathrm{cov}(p_1, p_2) \\ \mathrm{cov}(p_2, p_1) & \mathrm{var}(p_2) \end{bmatrix}$$

$$\mathrm{cov}(p, A) = G = \text{matrix} = \begin{bmatrix} \mathrm{cov}(p_1, g_1) & \mathrm{cov}(p_1, g_2) \\ \mathrm{cov}(p_2, g_1) & \mathrm{cov}(p_2, g_2) \end{bmatrix}$$

weights: $b = P^{-1} G v$

Index weights example

	σ_p	h^2	r_g	r_p	v	b
FW	.4	.4			5	2
			0	0		
FD	2	.4			-1	-0.4

Heritabilities same and no correlation;

Weights are proportional to rel. economic weight

Index weights example

	σ_p	h^2	r_g	r_p	v	b
FW	.4	.3			5	1.5
			0	0		
FD	2	.5			-1	-0.5

More weight for traits with higher heritability

Index weights example

	σ_p	h^2	r_g	r_p	v	b
FW	.4	.3			5	0.53
			0.5	0		
FD	2	.5			-1	-0.31

Weights also depend on correlations

In general, weights on phenotypic information sources are not easy to 'recognize'

Selection index for Single Trait

$$\mathrm{var}(p) = \mathrm{P} = \text{matrix} = \begin{bmatrix} \mathrm{var}(p_1) & \mathrm{cov}(p_1, p_2) \\ \mathrm{cov}(p_2, p_1) & \mathrm{var}(p_2) \end{bmatrix}$$

$$\mathrm{cov}(p,\mathrm{g}) = \mathrm{G} = \text{vector} = \begin{bmatrix} \mathrm{cov}(p_1, g) \\ \mathrm{cov}(p_2, g) \end{bmatrix}$$

weights: $\mathrm{b} = \mathrm{P}^{-1}\mathrm{G}$ gives weight for all sources about one EBV

Selection index for multiple traits

H = breeding objective = $v_1 g_1 + v_2 g_2$

var*(p)* = P = matrix = $\begin{bmatrix} \mathrm{var}(p_1) & \mathrm{cov}(p_1, p_2) \\ \mathrm{cov}(p_2, p_1) & \mathrm{var}(p_2) \end{bmatrix}$

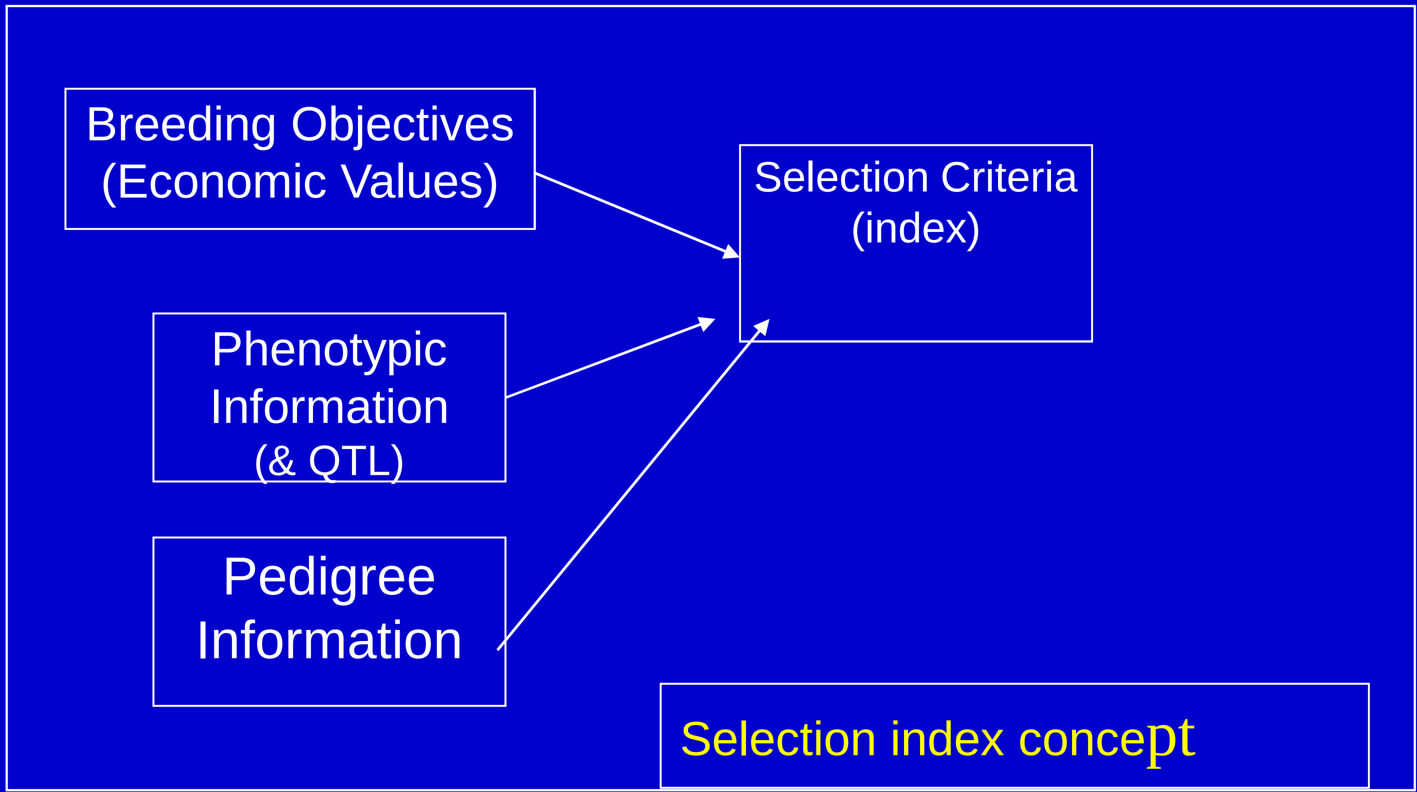
cov*(p,*g) = G = matrix = $\begin{bmatrix} \mathrm{cov}(p_1, g_1) & \mathrm{cov}(p_1, g_2) \\ \mathrm{cov}(p_2, g_1) & \mathrm{cov}(p_2, g_2) \end{bmatrix}$

weights: b = $P^{-1}Gv$ = $\begin{matrix} [b_{11} & b_{12}] \\ [b_{21} & b_{22}] \end{matrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$

b_i is a subset of weights for i^{th} trait to give EBV_i

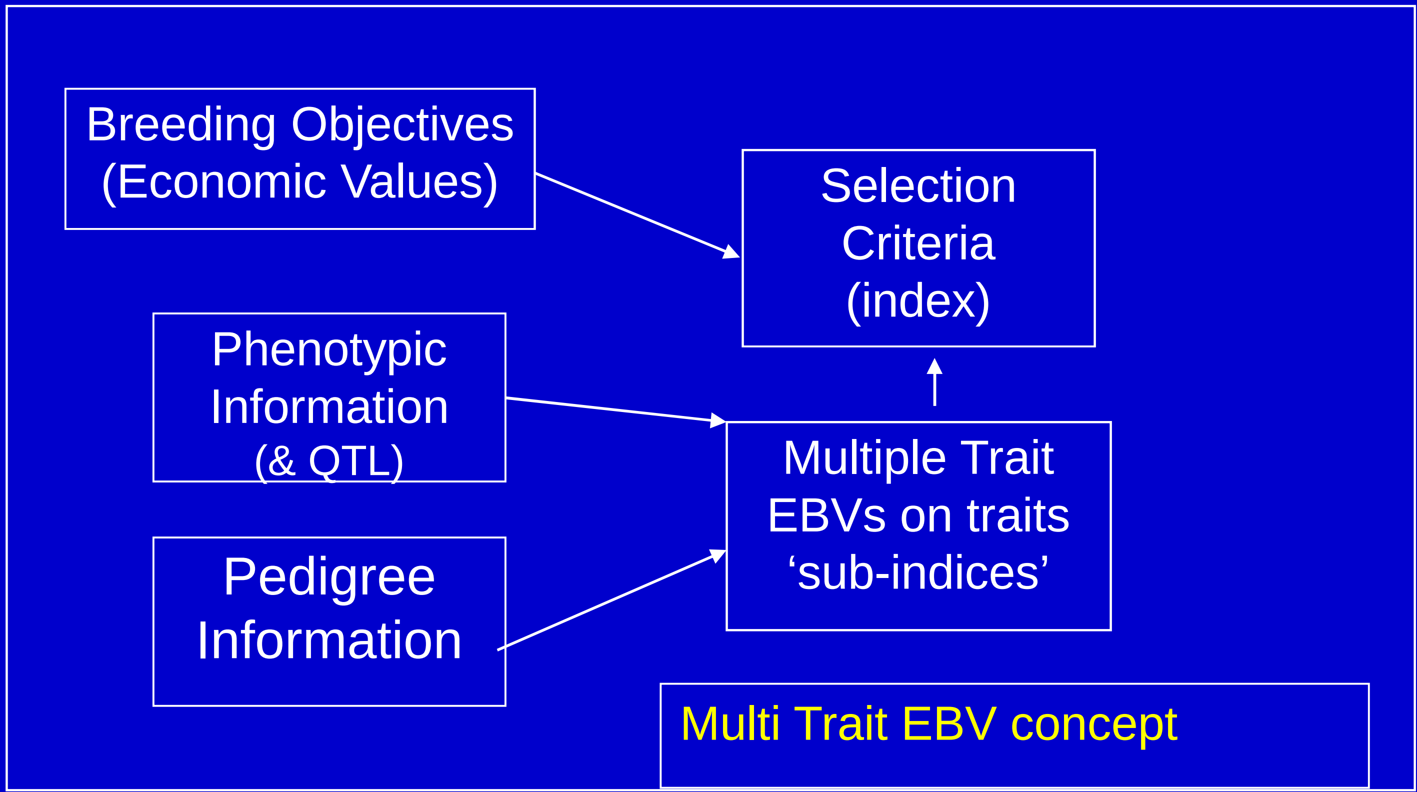
Overall weights are weighting each subset with its economic weight

Relationship between information, pedigree, index and multiple trait selection



$$\text{Index} = 0.195.W_{own} + 0.056.W_{sire} + 0.164.W_{HS} - 0.916.FI_{sire}$$

Relationship between information, pedigree, index and multiple trait selection



Index = $1.W_{EBV} - 10.FI_{EBV}$

Using EBV's rather than own phenotypes as selection criteria

$$\text{Index} = v_1\text{EBV}_1 + v_2\text{EBV}_2 + \ldots + v_n\text{EBV}_n$$

weights are equal to economic values!

as genetic parameters are already accounted for in MT-BLUP generation of EBV's

Index selection is more efficient than single trait selection!

Predicting genetic change to multiple trait selection

- Single trait selection response
- Correlated response to selection
- Response to index selection
  - How can multiple trait response be manipulated by varying index weights
  - Can we go anywhere we want?

Predicting Selection Response

- Total Response to selection (in $$)

$$R = i.r_{IH}.\sigma_A = i.\sigma_I \quad \text{in \$\$}$$

- Response for each trait (in trait units)

$$\delta g_i = b_{gi,I}\, R \qquad = i.b'G_i/\sigma_I$$

Regression of g_i on Index

See also mtindex.xls

Example

body weight	$h^2 = 0.40$	$\sigma_P = 17$ kg
feed intake	$h^2 = 0.25$	$\sigma_P = 2.0$ kg

r_g=.50 $\qquad$ r_p= 0.20

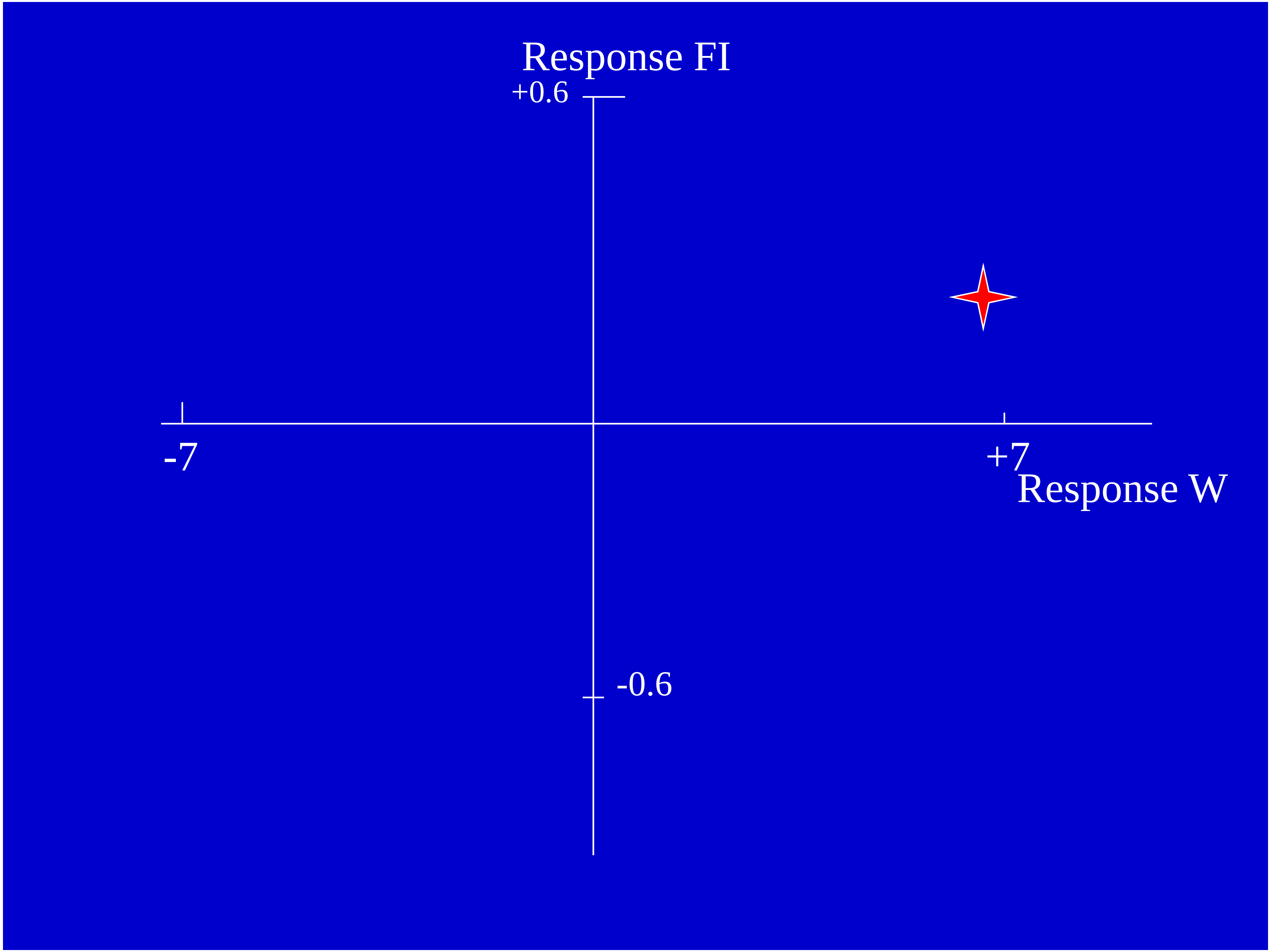
Criteria for selection | True Breeding Value

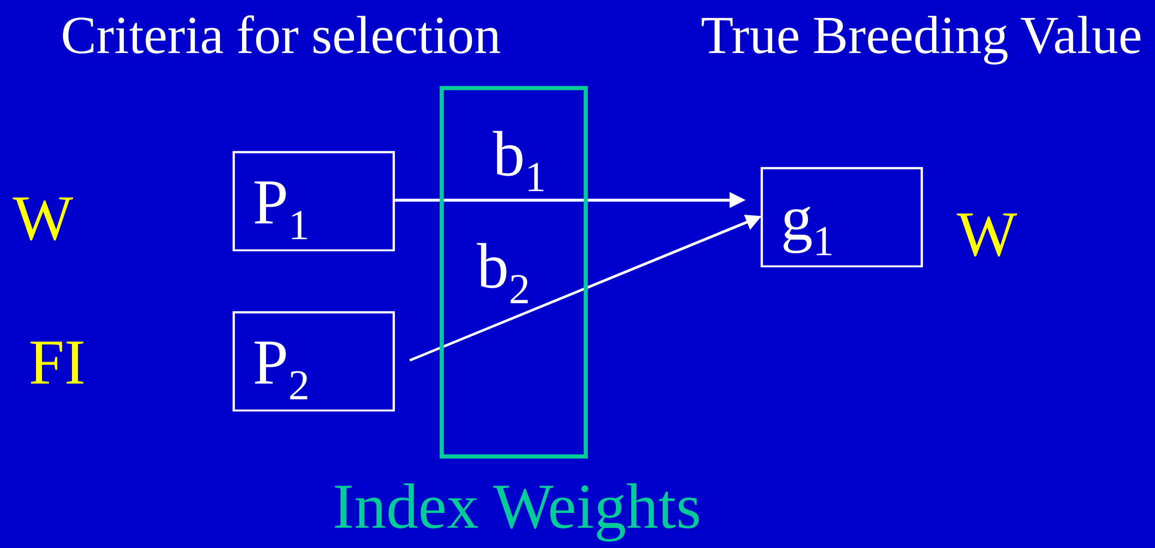
$$W \quad \boxed{P_1} \xrightarrow{b_1} \boxed{g_1} \quad W$$

Index = EBV = $0.4P_W$

Response = **6.80** kg Weight

Correl. Resp. = **0.32** kg Feed Intake


Response FI
+0.6
-7
+7
Response W
-0.6



$$\text{Index} = \$\text{EBV} = 0.38P_{W} + 0.69P_{FI}$$

R_W = **6.93** kg

R_{FI} = **0.40** kg

Combining information on two traits

breeding objective: H = g1 weight

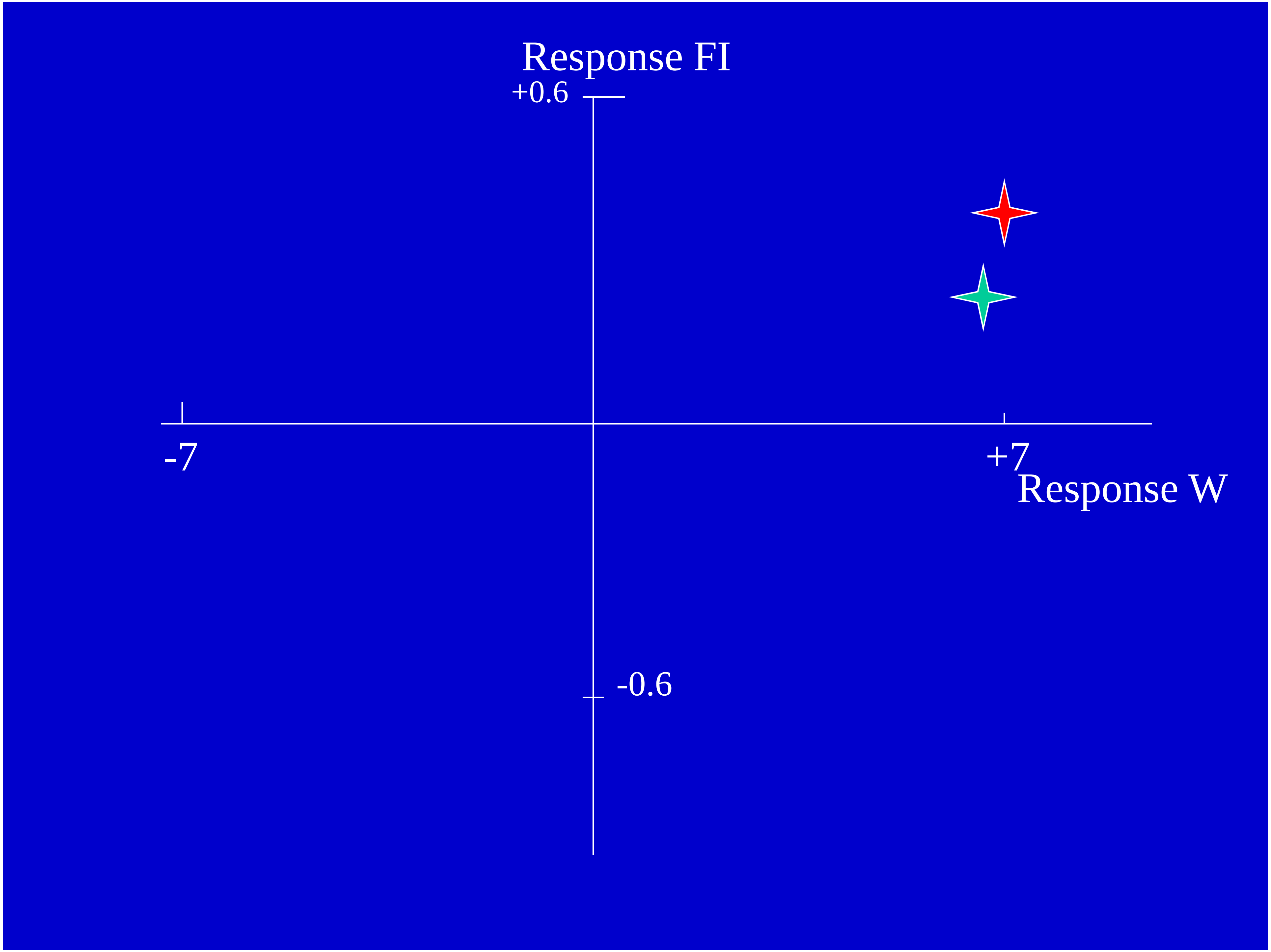
selection index $I = b_1X_1 + b_2X_2$

$$P = \mathrm{var}\begin{pmatrix} X_1 \\ X_2 \end{pmatrix} = \begin{pmatrix} \mathrm{var}(X_1) & \mathrm{cov}(X_1, X_2) \\ \mathrm{cov}(X_2, X_1) & \mathrm{var}(X_2) \end{pmatrix} = \begin{pmatrix} \sigma^2_{p_1} & r_p\sigma_{p_1}\sigma_{p_2} \\ r_p\sigma_{p_1}\sigma_{p_2} & \sigma^2_{p_2} \end{pmatrix}$$

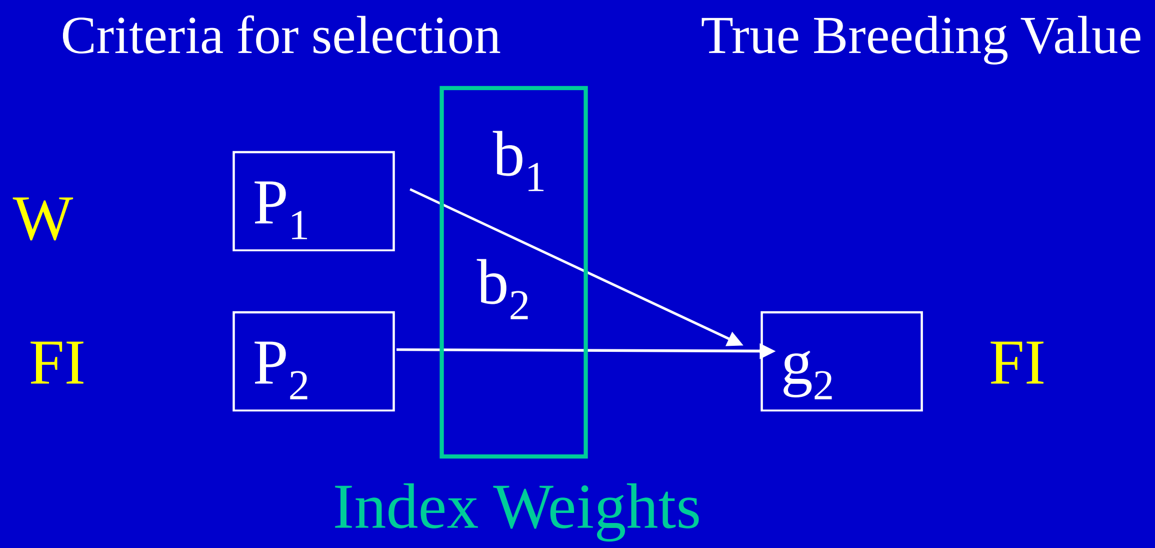
$$G = \mathrm{cov}\left(\begin{matrix} X_1 \\ X_2 \end{matrix}, A_1\right) = \begin{pmatrix} \mathrm{cov}(X_1, g_1) \\ \mathrm{cov}(X_2, g_1) \end{pmatrix} = \begin{pmatrix} \sigma^2_{A_1} \\ r_g\sigma_{A_1}\sigma_{A_2} \end{pmatrix}$$

$$b = P^{-1}G = \begin{pmatrix} 289 & 6.8 \\ 6.8 & 4 \end{pmatrix}^{-1} \begin{pmatrix} 115.6 \\ 0.692 \end{pmatrix} = \begin{pmatrix} 0.384 \\ 0.692 \end{pmatrix}$$

Index weights



Response FI
+0.6
-7
+7
Response W
-0.6

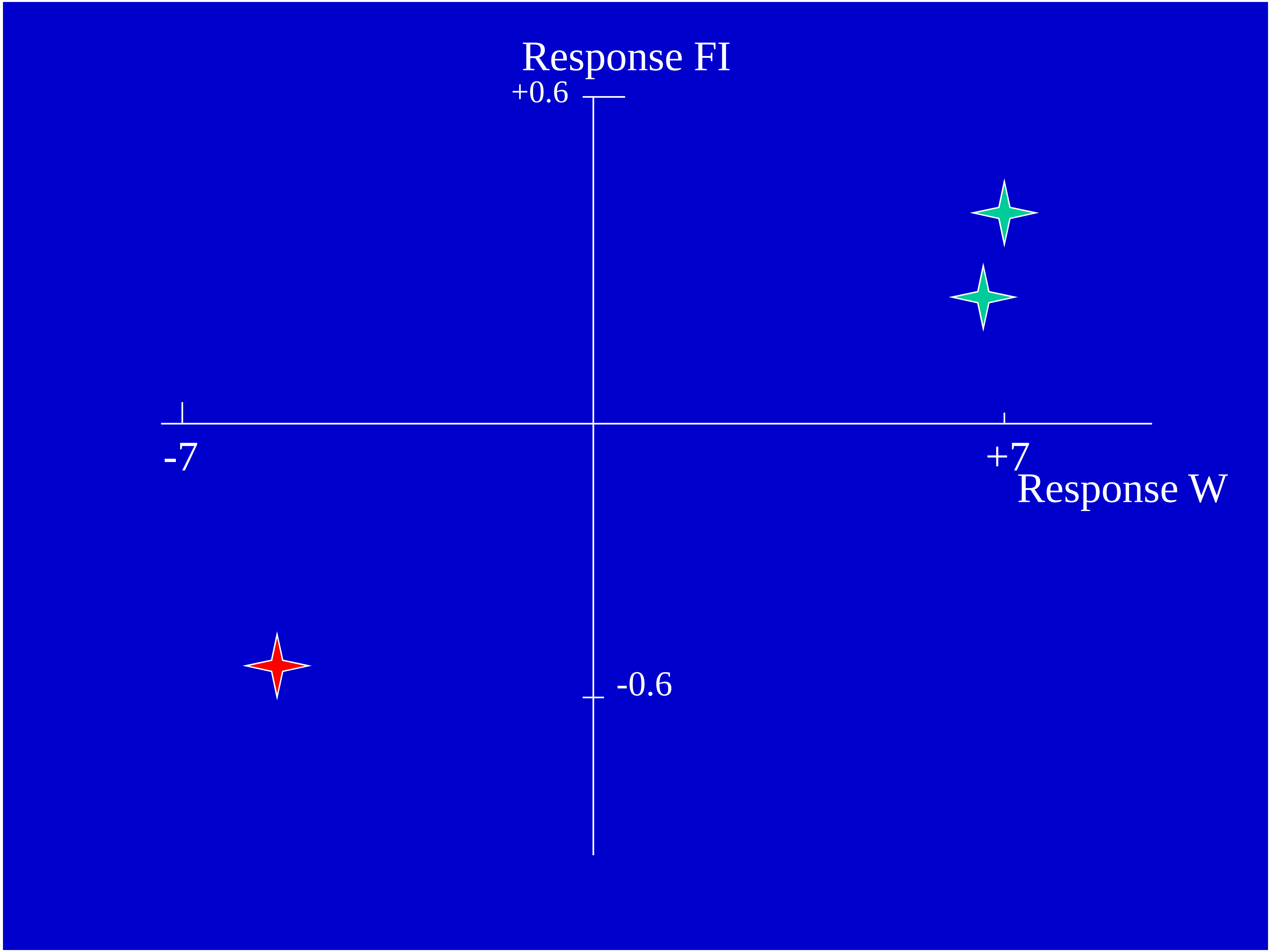


Selecting against Feed Intake

$$\text{Index} = \$\text{EBV} = -0.013P_{\text{W}} - 0.23P_{FI}$$

R_{W} = **-5.04** kg

R_{FI} = **-0.55** kg


Response FI
+0.6
-7
+7
Response W
-0.6

Multiple Trait breeding goal

weight feed intake

breeding objective: $H = v_1 g_1 + v_2 g_2$

assume first: $v_1 = 1$; $v_2 = -0.5$

selection index $I = b_1 X_l + b_2 X_2$

Multiple Trait breeding goal

- breeding objective: $H = v_1g_1 + v_2g_2$
- selection index $I = b_1X_I + b_2X_2$

$$G = \operatorname{cov}\left(\begin{pmatrix} X_1 \\ X_2 \end{pmatrix}, (g_1 \quad g_2)\right) = \begin{pmatrix} \operatorname{cov}(X_1, g_1) & \operatorname{cov}(X_1, g_2) \\ \operatorname{cov}(X_2, g_1) & \operatorname{cov}(X_2, g_2) \end{pmatrix}$$

$$= \begin{pmatrix} \sigma_{g_1}^2 & r_g\sigma_{A_1}\sigma_{A_2} \\ r_g\sigma_{A_1}\sigma_{A_2} & \sigma_{A2}^2 \end{pmatrix} = \begin{pmatrix} 5.75 & 2.74 \\ 2.74 & 14.5 \end{pmatrix}$$

$$\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = P^{-1}Gv = \begin{pmatrix} 289 & 6.8 \\ 6.8 & 4 \end{pmatrix}^{-1} \begin{pmatrix} 115.6 & 5.68 \\ 5.68 & 1 \end{pmatrix} \begin{pmatrix} 1.0 \\ -0.5 \end{pmatrix} = \begin{pmatrix} 0.37 \\ 0.66 \end{pmatrix}$$

weights

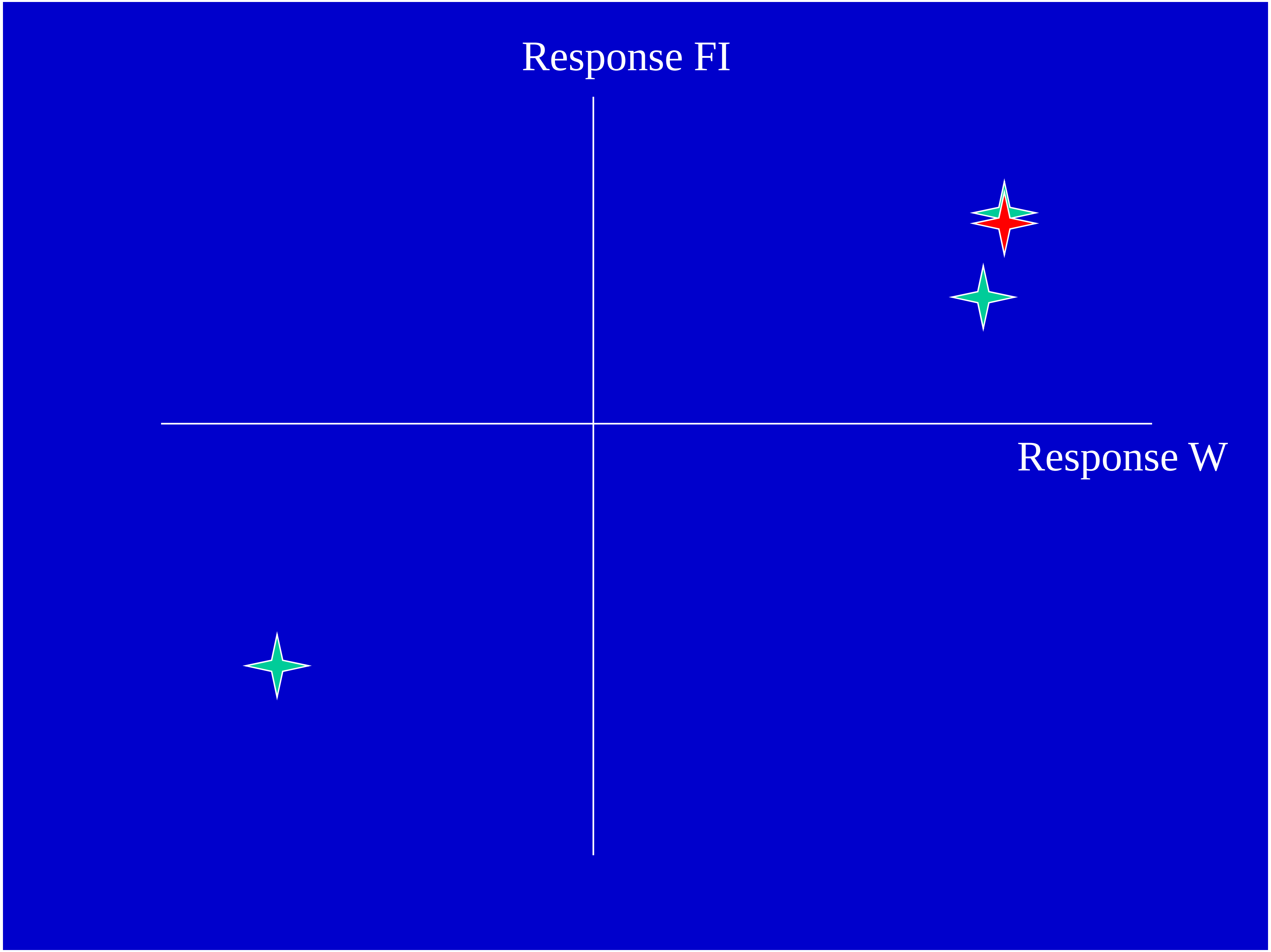
Criteria for selection | True Breeding Value

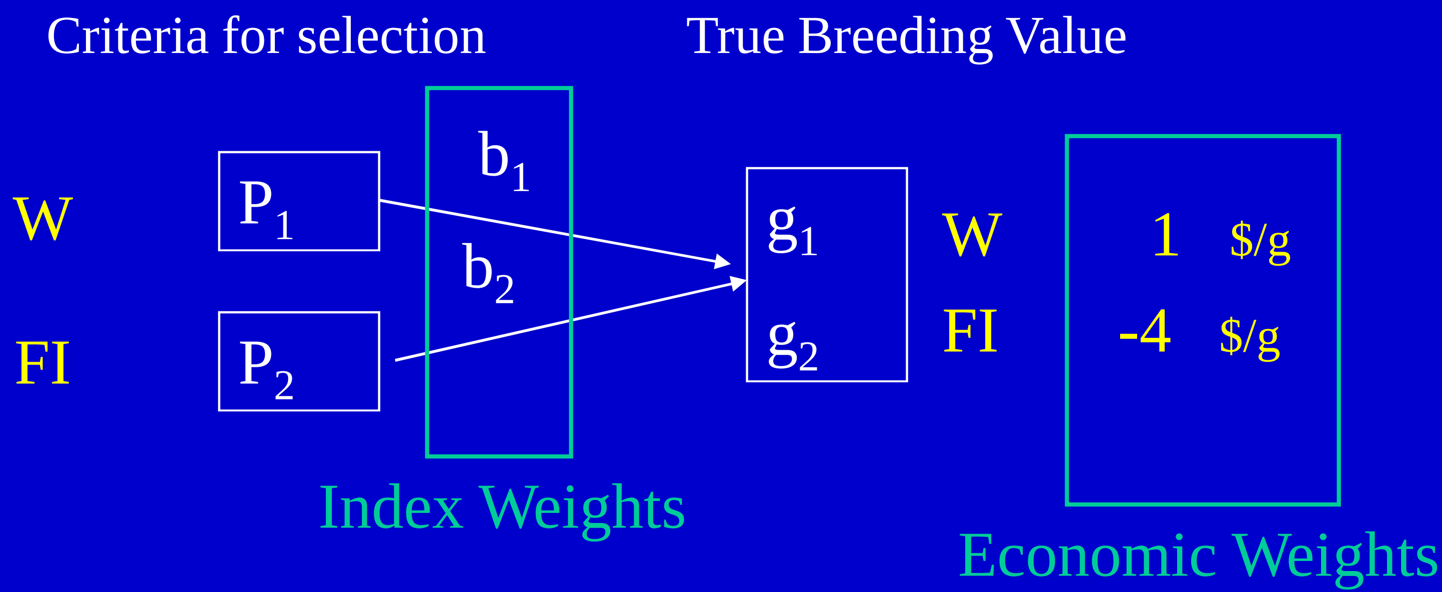
		Index Weights			Economic Weights
W	P_1	b_1	g_1	W	1 $/g
FI	P_2	b_2	g_2	FI	-0.5 $/g

Index = \$EBV = $0.38P_W$ +0.58

R_W = **6.93** kg

R_{FI} = **0.39** kg

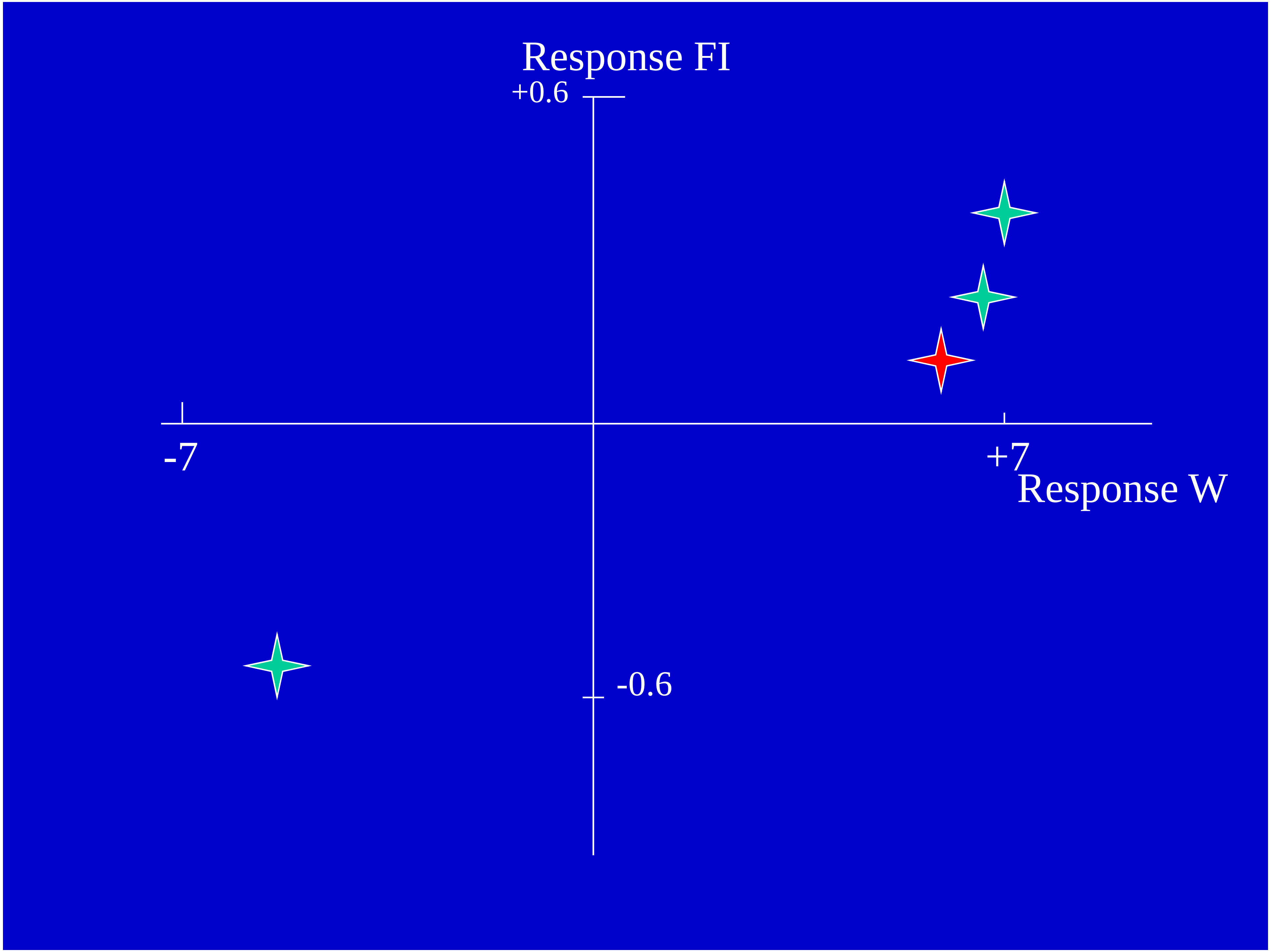

Response FI
Response W

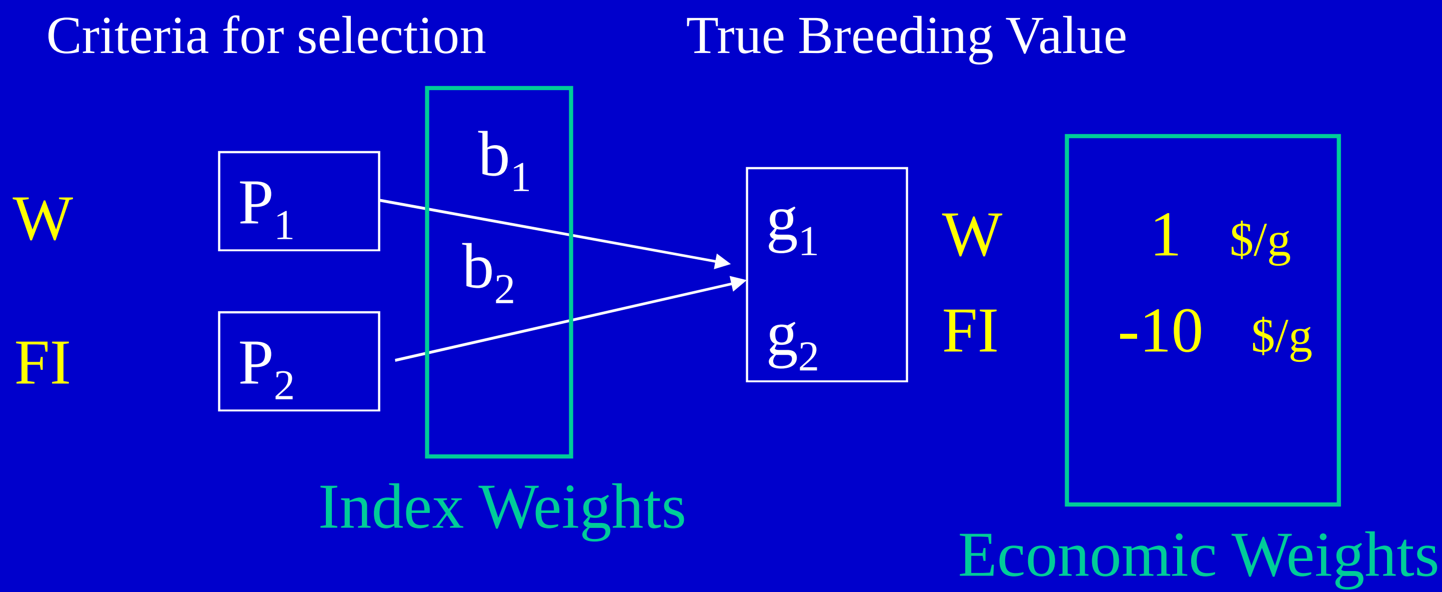


Index = \$EBV = $0.33P_W - 0.22P_{FI}$

R_W = **6.68** kg

R_{FI} = **0.28 k**g

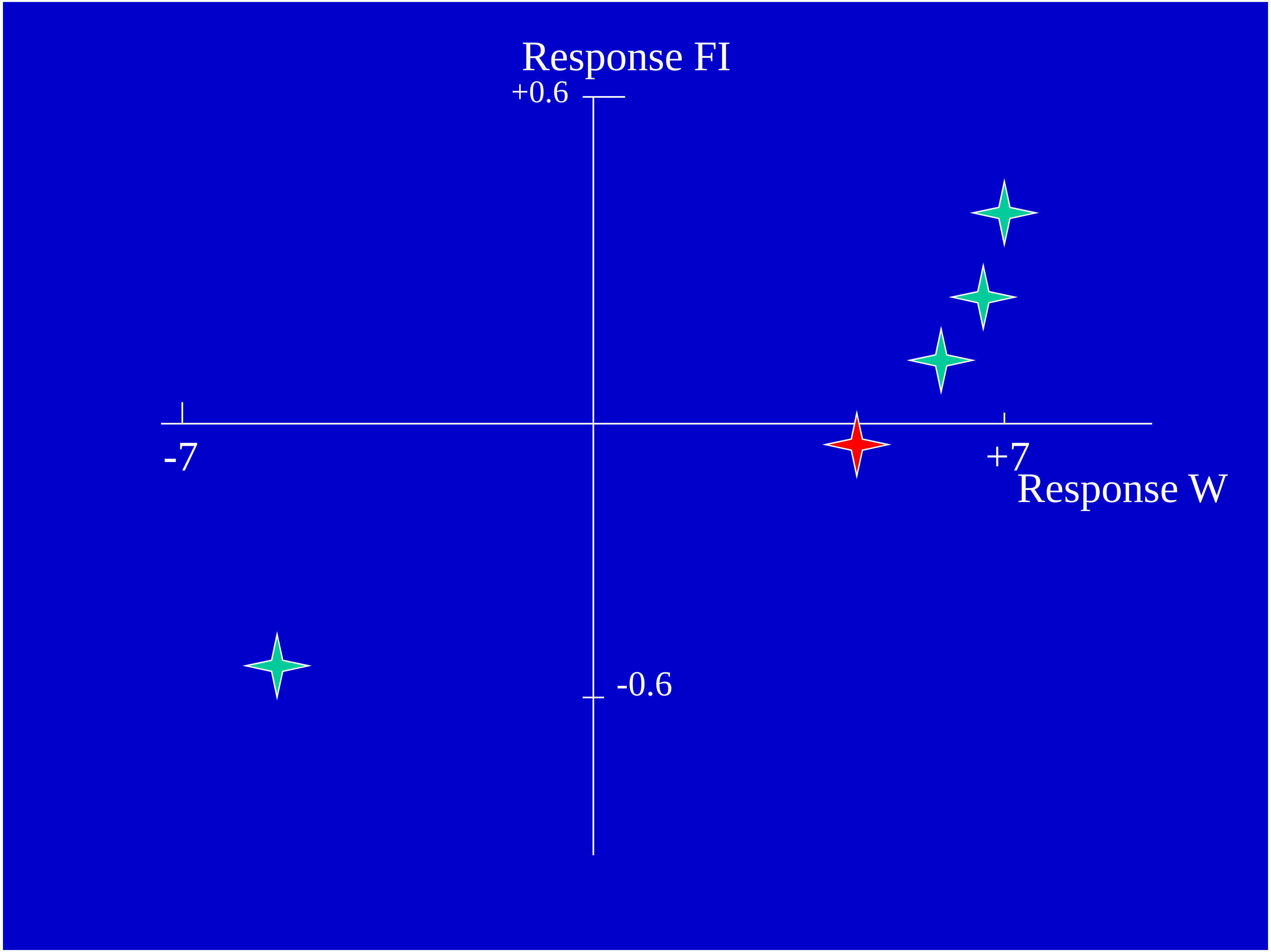

Response FI
+0.6
-7
+7
Response W
-0.6



$$\text{Index} = \$\text{EBV} = 0.25P_W - 1.58P_{FI}$$

R_W = **4.29** kg

R_{FI} = **-0.05 k**g

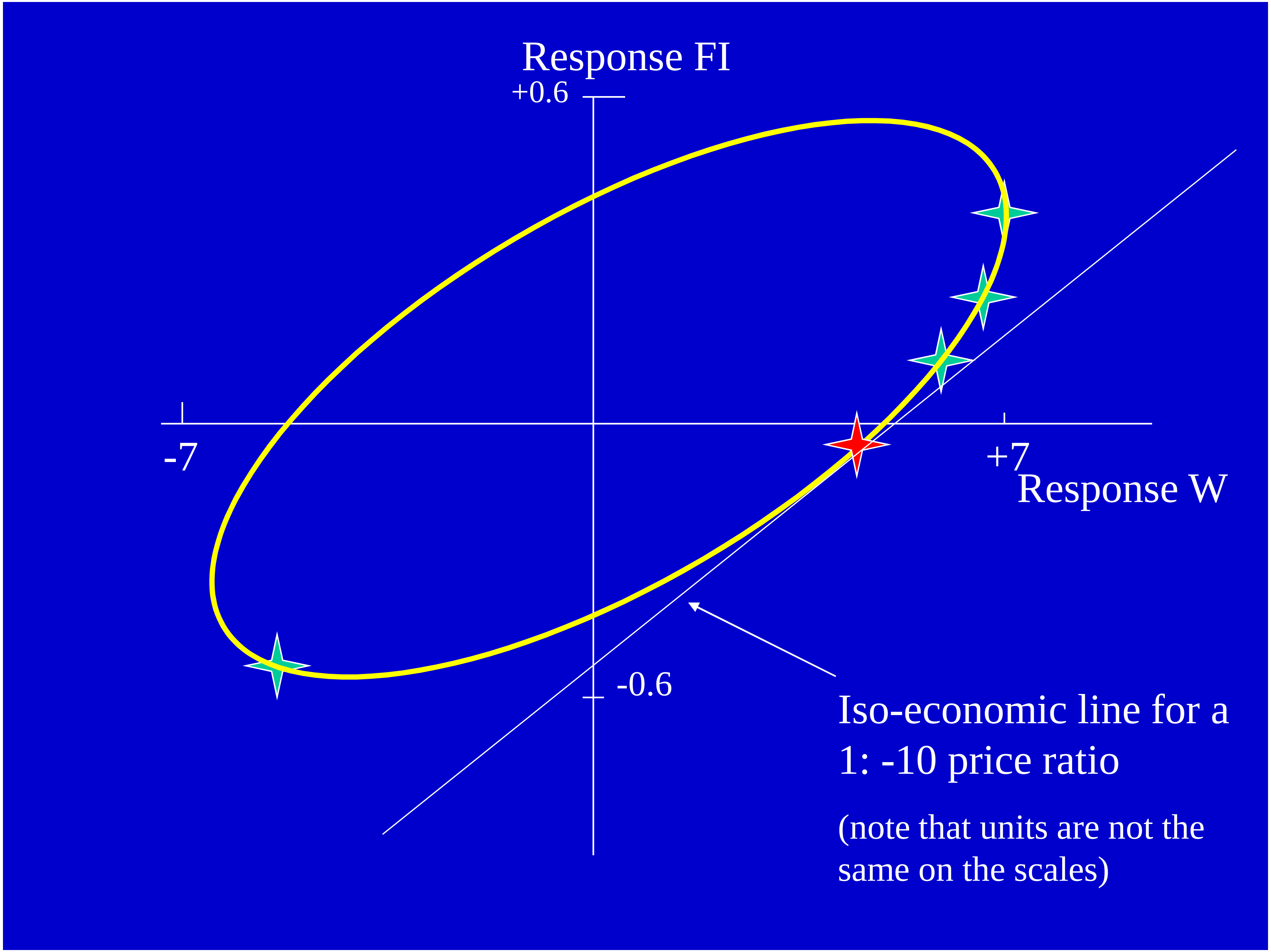

Response FI
+0.6
-7
+7
Response W
-0.6

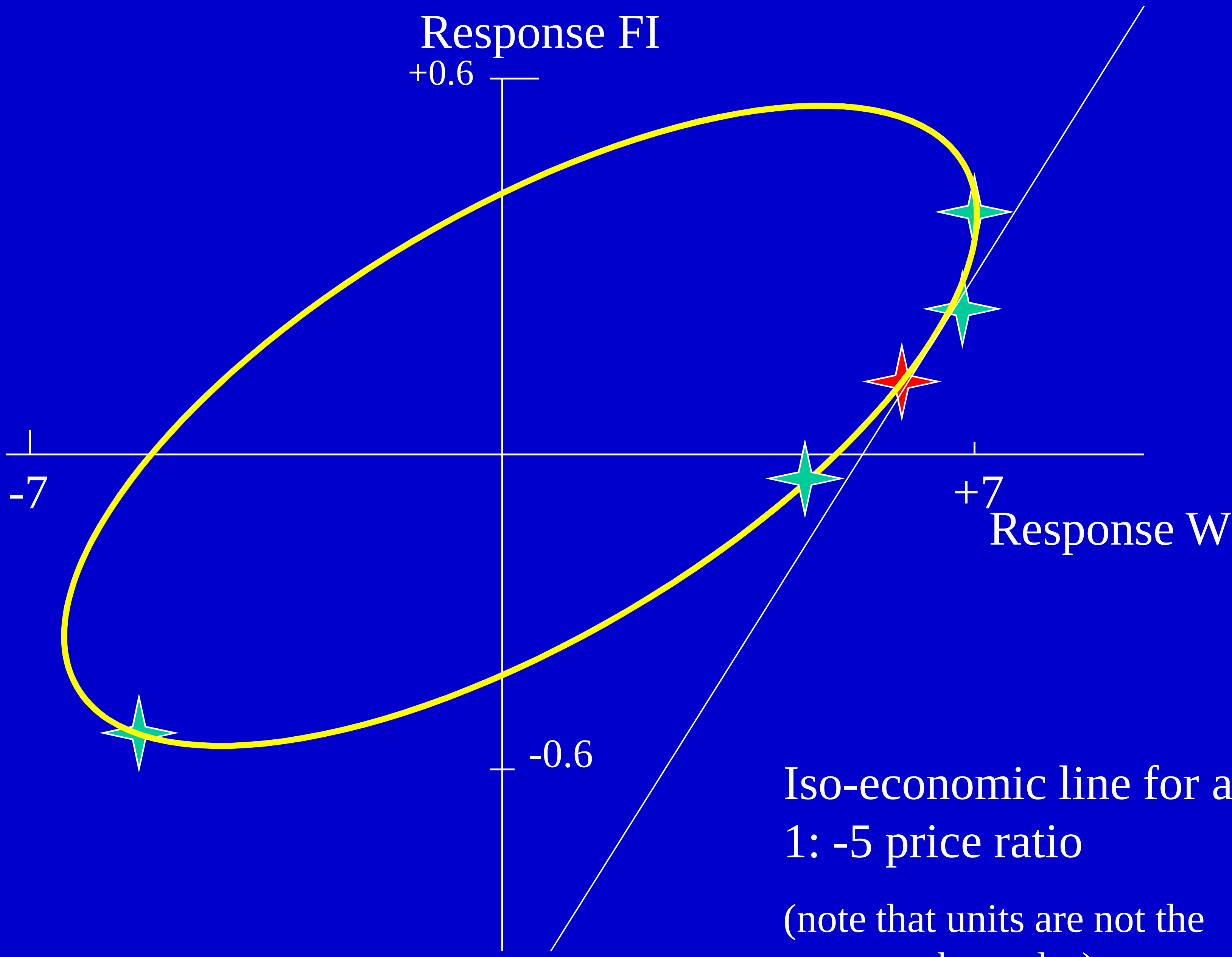
Summary of some possible responses

Information on	breeding goal		R. weight	R. feed intake
	a_1	a_2		
Weight	1	0	6.80	0.32
Feed	0	-1	-2.69	-0.50
Weight + feed	1	0	6.93	0.40
Weight + feed	0	-1	-5.64	-0.55
Weight + feed	0	-0.5	-5.93	0.39
Weight + feed	1	-1	6.92	0.38
Weight + feed	1	-4	6.68	0.28
Weight + feed	1	-10	4.29	-0.05
Weight + feed	1	-20	-0.93	-0.43

Note: Optimal selection pre-determined economic values,
response follows from that.

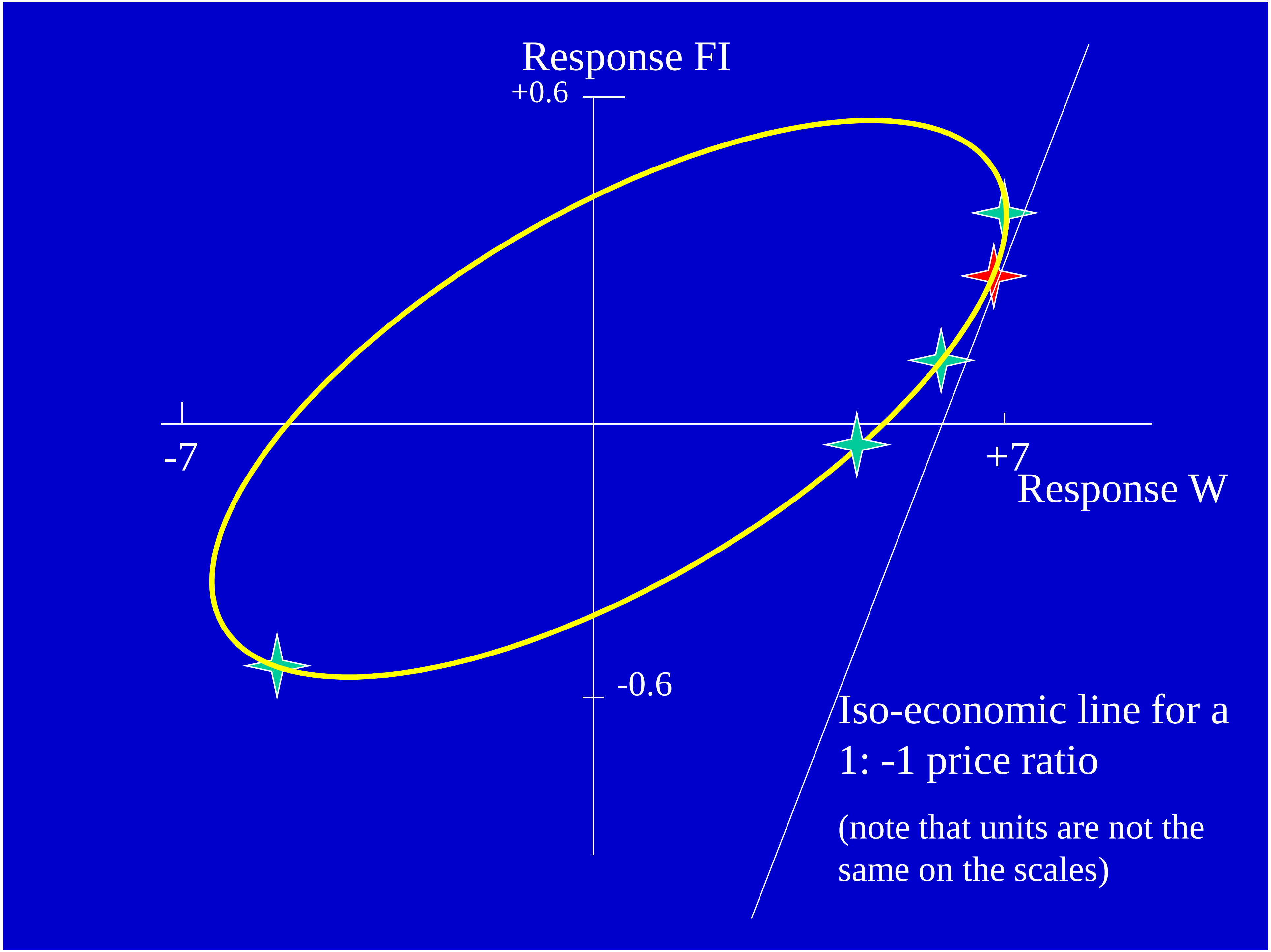
-otherwise: desired gains index
restricted index


Response FI
+0.6
-7
+7
Response W
-0.6
Iso-economic line for a 1: -10 price ratio
(note that units are not the same on the scales)



Iso-economic line for a 1: -5 price ratio

(note that units are not the same on the scales)


Response FI
+0.6
-7
+7
Response W
-0.6
Iso-economic line for a
1: -1 price ratio
(note that units are not the
same on the scales)

Predicting genetic change to multiple trait selection

- Response to index selection
  - How can multiple trait response be manipulated by varying index weights
  - Can we go anywhere we want?

mtindex.xls

Input Enter data only in light blue cells

4	Nr of traits						
			Parameters				
trait	Name	Units	Phenotypic Stand. Dev	Herit ability	Repeat ability	c2 among fulls sibs	Economic value
1	FW	kg	0.4	0.35	0	0	20
2	FD	mic	1.2	0.5	0	0	-30
3	Yield	%	5	0.25	0	0	0
4	BW	kg	3.5	0.4	0	0	0

Correlations

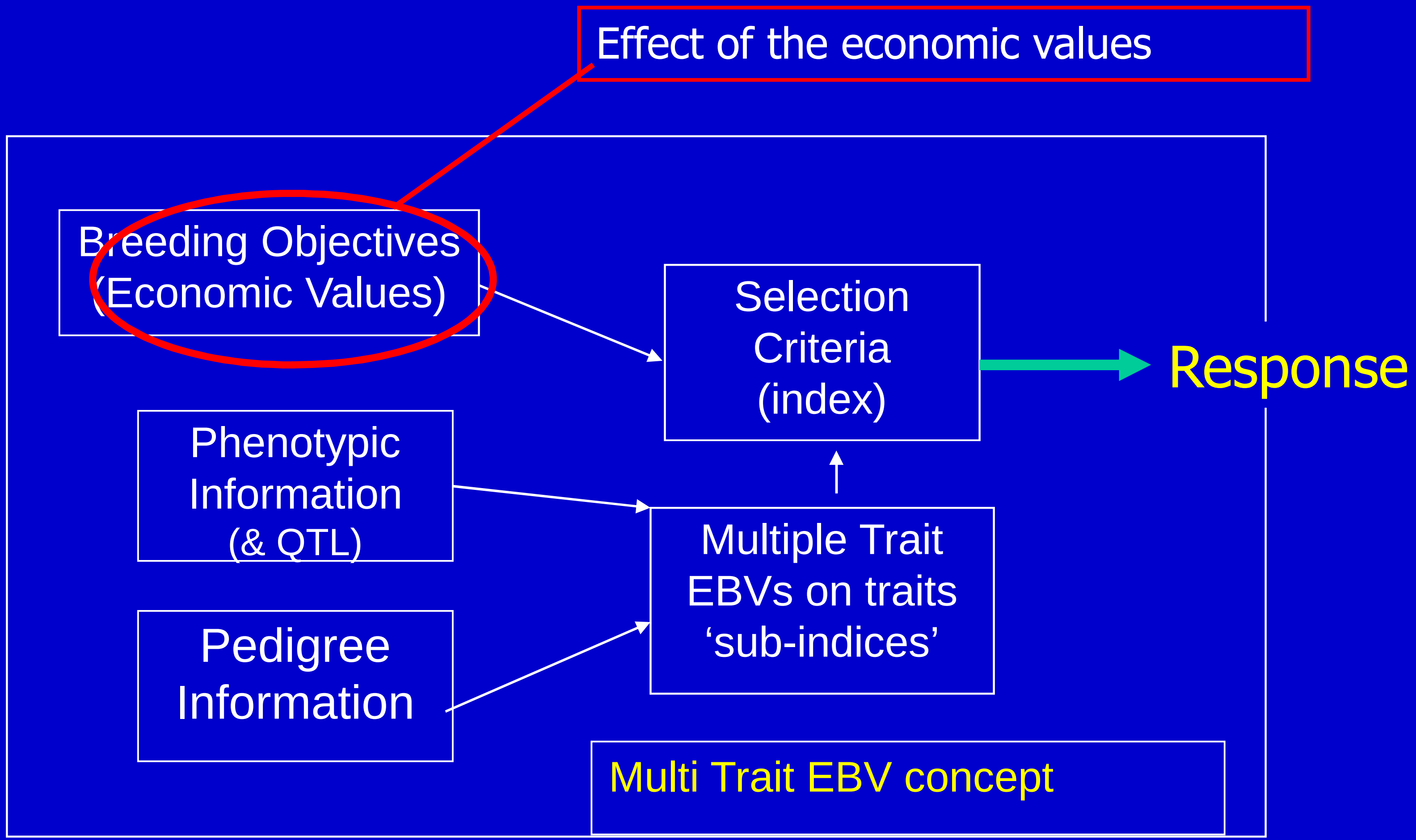
Correlation structure		1	2	3	4 Phenotypic
FW	1	1	0.3	0	0.3
FD	2	0.25	1	0	0.2
Yield	3	0	0	1	0
BW	4	0.4	0.2	0	1
0	Genetic				

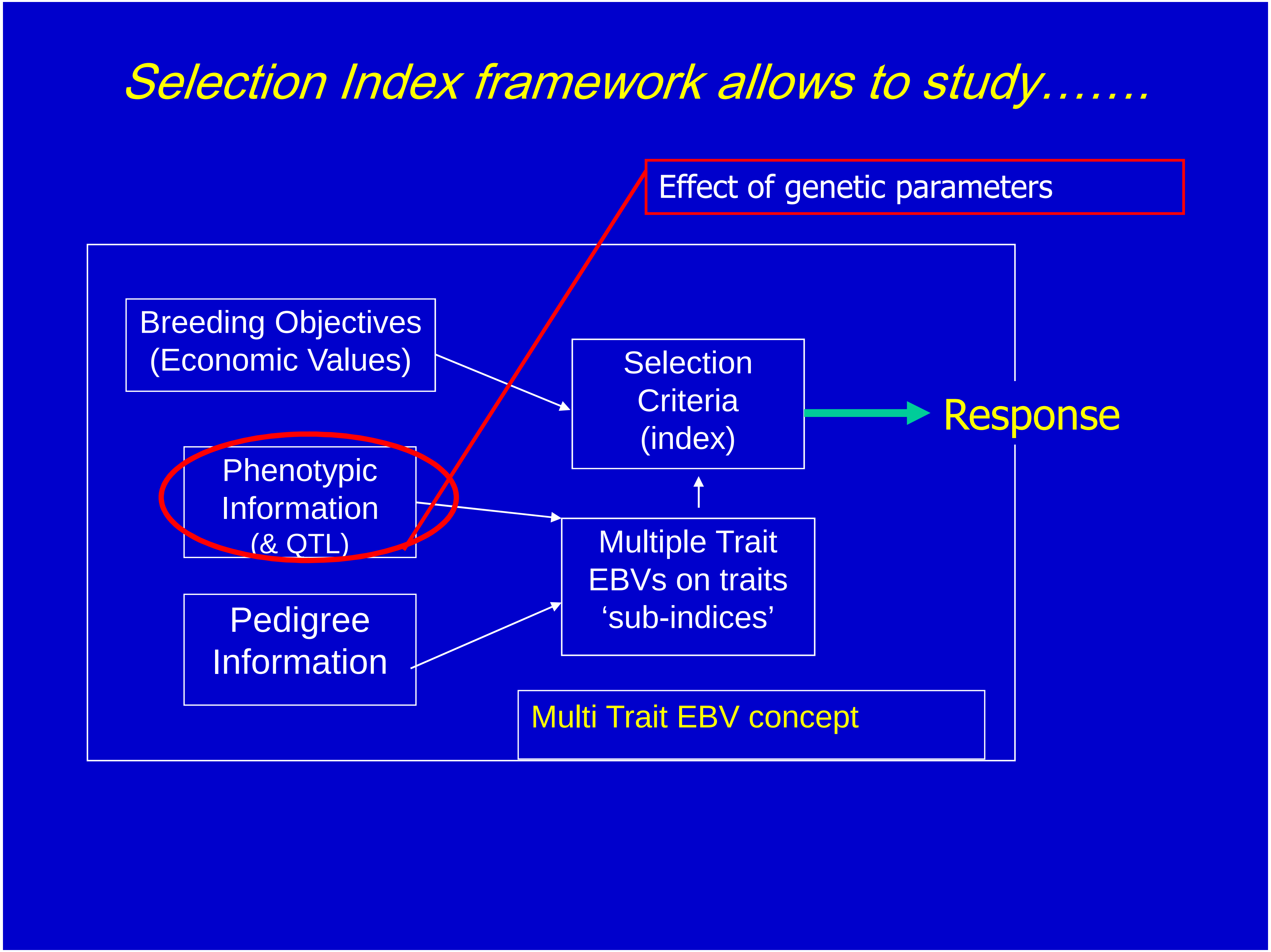
Results **Run**

SD Index	Accuracy of Index
19.340	0.7829

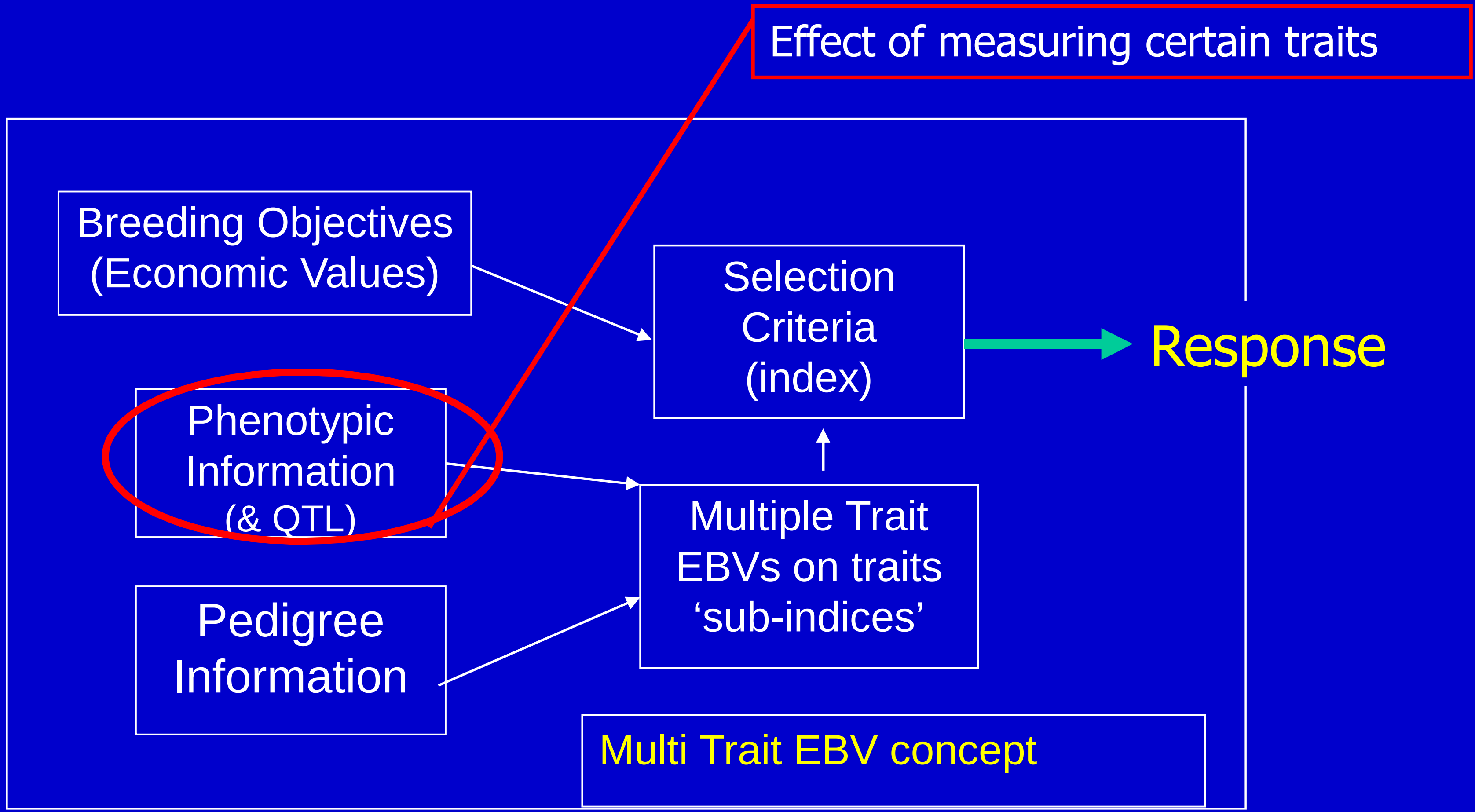
	Trait	Genetic Stand. Dev	**Response**		**Dollar value**	**Accuracy** MT-EBV	ST-EBV
1	FW	0.24	-0.01	kg	-0.19	0.699	0.696
2	FD	0.85	-0.65	mic	19.53	0.778	0.776
3	Yield	2.50	0.00	%	0.00	0.630	0.630
4	BW	2.21	-0.20	kg	0.00	0.726	0.725

Selection Index framework allows to study…….

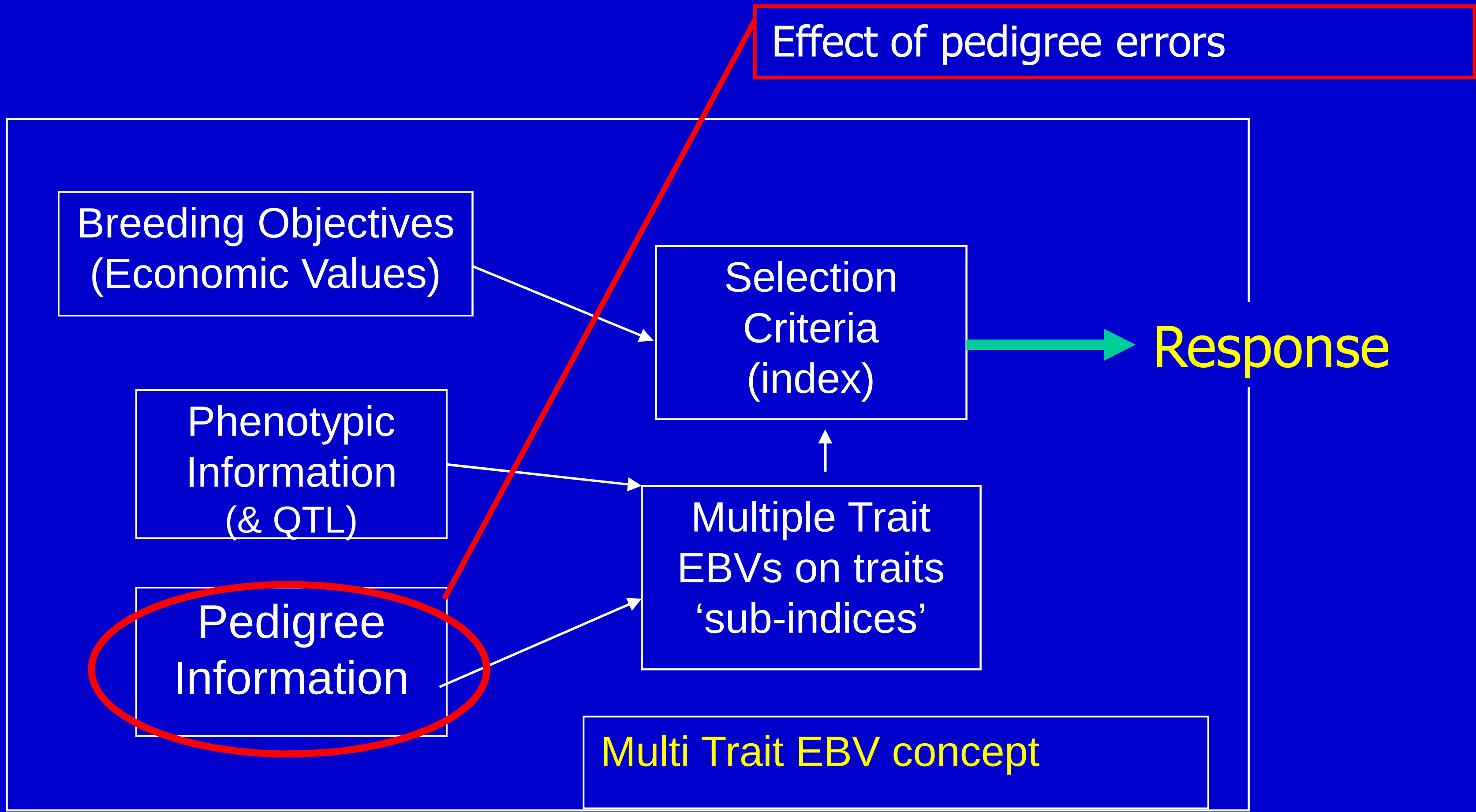



Selection Index framework allows to study…….
Effect of genetic parameters
Breeding Objectives
(Economic Values)
Selection
Criteria
(index)
Response
Phenotypic
Information
(& QTL)
Multiple Trait
EBVs on traits
'sub-indices'
Pedigree
Information
Multi Trait EBV concept

Selection Index framework allows to study…….



Selection Index framework allows to study…….



Are selection indices always linear?

- nonlinear profit function
- optimal traits
- threshold values for profit

Selection index with ‘desired gains’

- Rather than
  - determine econ. values >>>> response
  - We desire a response >>>> economic values (implicit)

When useful?