Breeding Objectives

Multi-trait selection - how to implement?

- Need to combine
  - the relative economic weights
  - genetic parameters (heritabilities, correlations)

\[ \text{Index} = b_1 X_1 + b_2 X_2 \]
Issues with MT selection

• We have to spread our selection efforts over several traits, each of them weighted economically

• Selection for one trait gives also a correlated response for other traits
We need weights for selection criteria

- Index = $b_1 X_1 + b_2 X_2 + \ldots + b_n X_n$
Selection index with more information sources (multiple regression)

\[ X = \text{vector with phenotypes (criteria)} \]
\[ A = \text{aggregate genotype (single trait here)} \]

\[
\begin{bmatrix}
\text{var}(X) = P = \text{matrix} = \\
\text{cov}(X, A) = G = \text{vector} =
\end{bmatrix}
\begin{bmatrix}
\text{var}(X_1) & \text{cov}(X_1, X_2) \\
\text{cov}(X_2, X_1) & \text{var}(X_2)
\end{bmatrix}
\]

\[
\begin{bmatrix}
\text{cov}(x_1, A) \\
\text{cov}(x_2, A)
\end{bmatrix}
\]

weights: \[ b = P^{-1}G \]
Selection index with more information sources and with more objective traits (multiple regression)

\[ X = \text{vector with phenotypes (criteria)} \]
\[ H = \text{aggregate genotype (multiple traits here)} \]
\[ = v_1 A_1 + v_2 A_2 \]
\[ \text{var}(X) = P = \text{matrix} = \begin{bmatrix} \text{var}(X_1) & \text{cov}(X_1, X_2) \\ \text{cov}(X_2, X_1) & \text{var}(X_2) \end{bmatrix} \]
\[ \text{cov}(X, A) = G = \text{matrix} = \begin{bmatrix} \text{cov}(X_1, A_1) & \text{cov}(X_1, A_2) \\ \text{cov}(X_2, A_1) & \text{cov}(X_2, A_2) \end{bmatrix} \]
\[ \text{weights: } b = P^{-1} G v \]
**Index weights example**

<table>
<thead>
<tr>
<th></th>
<th>$\sigma_p$</th>
<th>$h^2$</th>
<th>rg</th>
<th>rp</th>
<th>a</th>
<th>b</th>
</tr>
</thead>
<tbody>
<tr>
<td>FW</td>
<td>.4</td>
<td>.4</td>
<td>0</td>
<td>0</td>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>FD</td>
<td>2</td>
<td>.4</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>-0.4</td>
</tr>
</tbody>
</table>

Heritabilities same and no correlation;
Weights are proportional to rel. economic weight
More weight for traits with higher heritability
Index weights example

Weights also depend on correlations

In general, weights on phenotypic information sources are not easy to ‘recognize’

<table>
<thead>
<tr>
<th></th>
<th>σ_p</th>
<th>h^2</th>
<th>rg</th>
<th>rp</th>
<th>a</th>
<th>b</th>
</tr>
</thead>
<tbody>
<tr>
<td>FW</td>
<td>.4</td>
<td>.3</td>
<td>5</td>
<td>0</td>
<td>0.53</td>
<td></td>
</tr>
<tr>
<td>FD</td>
<td>2</td>
<td>.5</td>
<td>-1</td>
<td>0</td>
<td>-0.31</td>
<td></td>
</tr>
</tbody>
</table>
Selection index for Single Trait

\[ \text{var}(X) = P = \text{matrix} = \begin{bmatrix} \text{var}(x_1) & \text{cov}(x_1, x_2) \\ \text{cov}(x_2, x_1) & \text{var}(x_2) \end{bmatrix} \]

\[ \text{cov}(X, A) = G = \text{vector} = \begin{bmatrix} \text{cov}(x_1, A) \\ \text{cov}(x_2, A) \end{bmatrix} \]

weights: \( b = P^{-1}G \) gives weight for all sources about one EBV
Selection index for multiple traits

\[ H = \text{aggregate genotype} = v_1 A_1 + v_2 A_2 \]

\[ \text{var}(X) = P = \text{matrix} = \begin{bmatrix} \text{var}(x_1) & \text{cov}(x_1, x_2) \\ \text{cov}(x_2, x_1) & \text{var}(x_2) \end{bmatrix} \]

\[ \text{cov}(X,A) = G = \text{matrix} = \begin{bmatrix} \text{cov}(x_1, A_1) & \text{cov}(x_1, A_2) \\ \text{cov}(x_2, A_1) & \text{cov}(x_2, A_2) \end{bmatrix} \]

weights: \[ b = P^{-1} G v = [b_1 \quad b_2] \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \]

\[ b_i \text{ is a subset of weights for } i^{th} \text{ trait to give } \text{EBV}_i \]

Overall weights are weighting each subset with its economic weight.
Using EBV’s rather than own phenotypes as selection criteria

Index = \( v_1EBV_1 + v_2EBV_2 + \ldots + v_nEBV_n \)

weights are equal to economic values!
as genetic parameters are already accounted for in MT-BLUP generation of EBV’s

Index selection is more efficient than single trait selection!
Predicting response to MT selection

• Response in dollars:

\[ R = i \sigma_{\text{Index}} = i \sqrt{b'Pb} \]

Response for each trait

\[ [R_1 \ R_2 \ \ldots \ R_m] = i.b'G / \sqrt{b'Pb} \]
Are selection indices always linear?

- nonlinear profit function
- optimal traits
- threshold values for profit
Selection index with ‘desired gains’

• Rather than
  - determine econ. values  >>>>>  response

  - We desire a response  >>>>>  economic values (implicit)

  When useful?
Predicting genetic change to multiple trait selection

- Single trait selection response
- Correlated response to selection
- Response to index selection
  - How can multiple trait response be manipulated by varying index weights
  - Can we go anywhere we want?
Direct response to single trait selection

\[ R = i.h^2.\sigma_p \quad \text{if mass selection} \]

\[ R = i.r_{IA}.\sigma_A \quad \text{more general:} \]

\[ r_{IA} \text{ is accuracy of selection} \]
Direct and Correlated response to single trait selection

Response = $i.h_1.\sigma_{A1}$

and

Correlated Response = $i.h_1.r_g.\sigma_{A2}$
Combining information on two traits

selection index \[ I = b_1X_1 + b_2X_2 \]

\[
P = \begin{pmatrix} \text{var}(X_1) & \text{cov}(X_1, X_2) \\ \text{cov}(X_2, X_1) & \text{var}(X_2) \end{pmatrix} = \begin{pmatrix} \sigma_{p1}^2 & r_p \sigma_{p1} \sigma_{p2} \\ r_p \sigma_{p1} \sigma_{p2} & \sigma_{p2}^2 \end{pmatrix}
\]

\[
G = \begin{pmatrix} \text{cov}(X_1, A_1) \\ \text{cov}(X_2, A_1) \end{pmatrix} = \begin{pmatrix} \sigma_{A1}^2 \\ r_g \sigma_{A1} \sigma_{A2} \end{pmatrix}
\]

\[
b = P^{-1}G = \begin{pmatrix} 11.5 & 21.9 \\ 21.9 & 145 \end{pmatrix}^{-1} \begin{pmatrix} 5.75 \\ 2.74 \end{pmatrix} = \begin{pmatrix} 0.6517 \\ 0.0796 \end{pmatrix}
\]
Multiple Trait breeding goal

• Aggregate genotype: \(H = v_1 A_1 + v_2 A_2\)
• selection index \(I = b_1 X_1 + b_2 X_2\)
Multiple Trait breeding goal

- **Aggregate genotype:** \( H = v_1 g_1 + v_2 g_2 \)
- **selection index** \( I = b_1 X_1 + b_2 X_2 \)

\[
G = \text{cov} \left( \begin{pmatrix} X_1 \\ X_2 \end{pmatrix}, \begin{pmatrix} A_1 \\ A_2 \end{pmatrix} \right) = \begin{pmatrix} \text{cov}(X_1, A_1) & \text{cov}(X_1, A_2) \\ \text{cov}(X_2, A_1) & \text{cov}(X_2, A_2) \end{pmatrix}
\]

\[
= \begin{pmatrix} \sigma_{g_1}^2 & r_g \sigma_{A_1} \sigma_{A_2} \\ r_g \sigma_{A_1} \sigma_{A_2} & \sigma_{A_2}^2 \end{pmatrix} = \begin{pmatrix} 5.75 & 2.74 \\ 2.74 & 14.5 \end{pmatrix}
\]

\[
\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = P^{-1} Ga = \begin{pmatrix} 11.5 & 22.05 \\ 22.05 & 145 \end{pmatrix}^{-1} \begin{pmatrix} 5.75 & 2.74 \\ 2.74 & 14.5 \end{pmatrix} \begin{pmatrix} 1.0 \\ -0.5 \end{pmatrix} = \begin{pmatrix} 0.618 \\ -0.125 \end{pmatrix}
\]
• **Variance of index:**

\[ \sigma_I^2 = b'Pb = 3.27 \]  

• **Variance of the aggregate genotype**

\[ \sigma_H^2 = v'Cv = 6.64 \]  

\[ C = \text{var}(A) \text{ of breeding values} \]

• **Accuracy of Index:**

\[ r_{IH} = \frac{\sigma_I}{\sigma_H} = \sqrt{\frac{b'Pb}{v'Cv}} \]
• **Response to selection**

\[ R = i.r_{ih} \cdot \sigma_A = i.\sigma_i \quad \text{in } \$$ \]

• **Response in each trait:**

\[ \delta g_i = b_{gi,l} R \]
\[ = i.b'G_i/\sigma_i \]

• **Notice that sum of**

\[ \delta g_i \cdot v_i = R \]

See also mtindex.xls
Example

body weight  \( h^2_A = 0.40 \quad \sigma_P = 17 \text{ kg} \)

feed intake  \( h^2_B = 0.25 \quad \sigma_P = 2.0 \text{ kg} \)

\( r_g = 0.50 \quad r_p = 0.20 \)

selection intensity = 1.0
Criteria for selection

True Breeding Value

\[ W \quad P_1 \quad b_1 \quad A_1 \quad W \]

Index = EBV = 0.4\( P_w \)

Response = 6.80 kg Weight

Correl. Resp. = 0.32 kg Feed Intake
Index = $EBV = 0.38P_W + 0.69P_{FI} \quad R_W = 6.93 \text{ kg} \quad R_{FI} = 0.40 \text{ kg}
Criteria for selection

True Breeding Value

\[ W \]

\[ P_1 \]

\[ b_1 \]

Index Weights

\[ FI \]

\[ P_2 \]

\[ b_2 \]

\[ A_2 \]

\[ FI \]

\[ \text{Index} = \$EBV = -0.013P_W - 0.23P_{FI} \]

\[ R_W = -5.04 \text{ kg} \]

\[ R_{FI} = -0.55 \text{ kg} \]
Index = $EBV = 0.62P_W - 0.13P_{FI}$

$R_W = 6.93$ kg

$R_{FI} = 0.39$ kg
Criteria for selection

True Breeding Value

\[ \text{Index} = \$EBV = 0.33P_W - 0.22P_{FI} \]

### Economic Weights

- \( W_{FI} = 0.28 \text{ kg} \)
- \( W_{FI} = 6.68 \text{ kg} \)

### Index Weights

- \( W_{FI} = 1 \text{ $/g} \)
- \( W_{FI} = -4 \text{ $/g} \)
Index $\text{EBV} = 0.25P_w - 1.58P_{FI}$

$R_w = 4.29 \text{ kg}$

$R_{FI} = -0.05 \text{ kg}$
**Summary of some possible responses**

<table>
<thead>
<tr>
<th>Information on</th>
<th>breeding goal</th>
<th>R. weight</th>
<th>R. feed intake</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$a_1$</td>
<td>$a_2$</td>
<td></td>
</tr>
<tr>
<td>Weight</td>
<td>1</td>
<td>0</td>
<td>6.80</td>
</tr>
<tr>
<td>Feed</td>
<td>0</td>
<td>-1</td>
<td>-2.69</td>
</tr>
<tr>
<td>Weight + feed</td>
<td>1</td>
<td>0</td>
<td>6.93</td>
</tr>
<tr>
<td>Weight + feed</td>
<td>0</td>
<td>-1</td>
<td>-5.64</td>
</tr>
<tr>
<td>Weight + feed</td>
<td>0</td>
<td>-0.5</td>
<td>-5.93</td>
</tr>
<tr>
<td>Weight + feed</td>
<td>1</td>
<td>-1</td>
<td>6.92</td>
</tr>
<tr>
<td>Weight + feed</td>
<td>1</td>
<td>-4</td>
<td>6.68</td>
</tr>
<tr>
<td>Weight + feed</td>
<td>1</td>
<td>-10</td>
<td>4.29</td>
</tr>
<tr>
<td>Weight + feed</td>
<td>1</td>
<td>-20</td>
<td>-0.93</td>
</tr>
</tbody>
</table>

**Note:** Optimal selection pre-determined economic values, response follows from that.

-otherwise: desired gains index
restricted index
Iso-economic line for a 1: -10 price ratio

(note that units are not the same on the scales)
Iso-economic line for a 1: -5 price ratio
(note that units are not the same on the scales)
Iso-economic line for a 1: -1 price ratio

(note that units are not the same on the scales)
Predicting genetic change to multiple trait selection

• Response to index selection
  – How can multiple trait response be manipulated by varying index weights
  – Can we go anywhere we want?