Evaluation of animals in practice

- Need proper data (centralized)
  - recording system (management groups)
  - correct animal identification
  - other issues?

- Need proper model
  - Account for bias and selection
  - Account for other effects (maternal, permanent environment, multiple trait, different breeds)

A data collection system

- Rules for recording
- Rules for invalid data
- Avoid selective recording
- Rules for defining management groups
- Impetus for doing the right thing
  - What are mechanisms for regulating the system?

Choosing the right model

- Technical issues
  - How to implement the different effects (see next)

- Political issues
  - Evaluation (and reporting) across herds
  - Evaluation across breeds
  - Evaluation across countries
  - Finding a balance between individual and collective satisfaction

More political issues

– Price for the services
– Ownership of the genetic evaluation results
– Ownership of data
– …..

How to expand the simple mixed model

- Simple mixed model

  y = contempgrp + animal + residual

- More general
  - y = fixed effects + random effects + residual

fixed effects	random effects	residual
• cg • age • …..	• animal • maternal • permanent env.	homo/ heterogeneous

$$\mathbf{y} = \mathbf{Xb} + \mathbf{Zu} + \mathbf{e}$$

A sire model

$$\mathbf{y = \quad Xb \quad + \quad Zs \quad + \quad \varepsilon}$$

$$\text{var(u)} = \text{A}\ \sigma_s^2$$

$$\text{var(e)} = \text{I}\ \sigma_\varepsilon^2$$

$$\begin{bmatrix} X'X & X'Z \\ Z'X & Z'Z + \lambda A^{-1} \end{bmatrix} \begin{bmatrix} b \\ s \end{bmatrix} = \begin{bmatrix} X'y \\ Z'y \end{bmatrix}$$

$$\lambda = \sigma_\varepsilon^2 / \sigma_s^2$$

originally used (pre-1985)

fewer equations for amount of data

ignores dam-side

Some formal definitions of the model

$$\mathbf{y} = \mathbf{Xb} + \mathbf{Zu} + \mathbf{e}$$

var(u) = G

var(e) = R

var(y) = ZGZ’ + R

$$\begin{bmatrix} X'R^{-1}X & X'R^{-1}Z \\ Z'R^{-1}X & Z'R^{-1}Z + G^{-1} \end{bmatrix} \begin{bmatrix} b \\ u \end{bmatrix} = \begin{bmatrix} X'R^{-1}y \\ Z'R^{-1}y \end{bmatrix}$$

simple version

$$\text{var(u)} = A\ \sigma_a^2$$

$$\text{var(e)} = I\ \sigma_e^2$$

$$\begin{bmatrix} X'X & X'Z \\ Z'X & Z'Z + \lambda A^{-1} \end{bmatrix} \begin{bmatrix} b \\ u \end{bmatrix} = \begin{bmatrix} X'y \\ Z'y \end{bmatrix}$$

A simple example of variance structure

animal	obs'n
1	9
1	11
2	10

$$Z = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$Z'Z = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \qquad ZZ' = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$G = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \sigma_{anm}^2$$

$$\text{var(y)} = \text{ZGZ'} + \text{R} \quad = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \sigma_{anm}^2 + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \sigma_e^2$$

Repeatability model

y = Xb + Zu + Zp + ε

$$G = \begin{pmatrix} A\sigma_a^2 & 0 \\ 0 & I\sigma_c^2 \end{pmatrix} \qquad \mathrm{var}\begin{pmatrix} u \\ p \\ e \end{pmatrix} = \begin{pmatrix} A\sigma_a^2 & 0 & 0 \\ 0 & I\sigma_c^2 & 0 \\ 0 & 0 & I\sigma_e^2 \end{pmatrix} = \begin{pmatrix} \mathrm{G} & 0 \\ 0 & \mathrm{R} \end{pmatrix}$$

$$\begin{pmatrix} X'X & X'Z & X'Z \\ Z'X & Z'Z + \alpha A^{-1} & Z'Z \\ Z'X & Z'Z & Z'Z + \gamma I \end{pmatrix} \begin{pmatrix} b \\ u \\ p \end{pmatrix} = \begin{pmatrix} X'y \\ Z'y \\ Z'y \end{pmatrix}$$

Maternal effects model

$$\mathbf{y} = \mathbf{Xb} + \mathbf{Z_1u} + \mathbf{Z_2m} + \boldsymbol{\varepsilon}$$

Direct animal effect

Maternal effect

$$G = \begin{pmatrix} A\sigma_a^2 & A\sigma_{am} \\ A\sigma_{am} & A\sigma_{mc}^2 \end{pmatrix}$$

Covariance

$$\operatorname{var}\begin{pmatrix} u \\ m \\ e \end{pmatrix} = \begin{pmatrix} A\sigma_a^2 & A\sigma_{am} & 0 \\ A\sigma_{am} & A\sigma_m^2 & 0 \\ 0 & 0 & I\sigma_e^2 \end{pmatrix}$$

$$\begin{pmatrix} X'X & X'Z_1 & X'Z_2 \\ Z_1'X & Z_1'Z_1 + \alpha_{11}A^{-1} & Z_1'Z_2 + \alpha_{12}A^{-1} \\ Z_2'X & Z_2'Z_1 + \alpha_{21}A^{-1} & Z_2'Z_2 + \alpha_{22}A^{-1} \end{pmatrix} \begin{pmatrix} b \\ u \\ m \end{pmatrix} = \begin{pmatrix} X'y \\ Z_1'y \\ Z_2'y \end{pmatrix}$$

Consequence of more complex models

- Usually (many) more equations
- Do we know the parameters (variance components)?
- More difficult to interpret results
- Often more accurate (and less biased)
  - Account for maternal effects
  - Account for heterogeneous variance (animals maybe be more different in some herds/flocks than in other)

Genetic groups

- Consider them as fixed effect in the model
- But add those to breeding values........

- **$EBV_{across} = EBV_{within} + group\_solution$**

- Grouping needed whenever there is a **genetic** difference in base animals
  - (to account for selection: breeds, origin,....)
- Only need to group the unknown parents
  - Remember that relationships matrix accounts for other selection

Example of genetic groups

Michael Angus	315	Mean Angus	300
Whiskey Hereford	315	Mean Hereford	320

	EBV_{within}	EBV_{across}
Michael Angus	+ 6	+ 6
Whiskey Hereford	- 2	+ 18